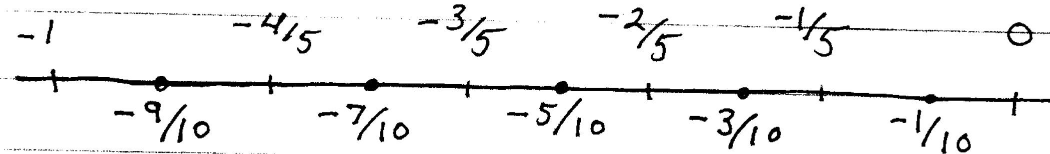
Section 7.5

2.) $\int_{-1}^{0} (x+1)^3 dx$, $n=5$, $f(x) = (x+1)^3$



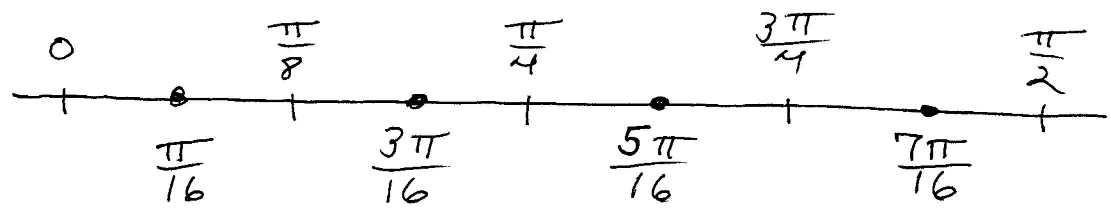
$h = \frac{0-(-1)}{5} = 1/5$ so Midpoint Estimate

$= \frac{1}{5}\left[f\left(\frac{-9}{10}\right) + f\left(\frac{-7}{10}\right) + f\left(\frac{-5}{10}\right) + f\left(\frac{-3}{10}\right) + f\left(\frac{-1}{10}\right)\right]$

$= \frac{1}{5}\left[\frac{1}{1000} + \frac{27}{1000} + \frac{125}{1000} + \frac{343}{1000} + \frac{729}{1000}\right]$

$= \frac{1}{5} \cdot \frac{1225}{1000} = 0.245$

4.) $\int_{0}^{\frac{\pi}{2}} \sin x \, dx$, $n=4$, $f(x) = \sin x$:



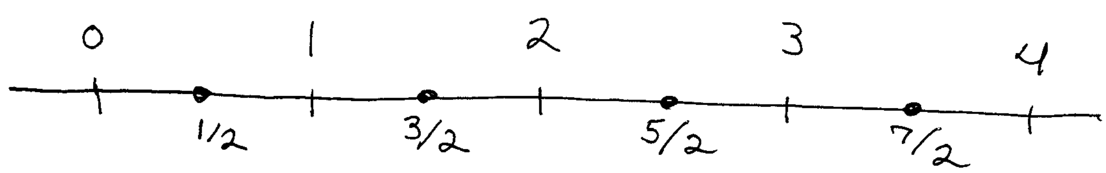
$h = \frac{\frac{\pi}{2} - 0}{4} = \frac{\pi}{8}$ so Midpoint Estimate

$M_4 = \frac{\pi}{8}\left[f\left(\frac{\pi}{16}\right) + f\left(\frac{3\pi}{16}\right) + f\left(\frac{5\pi}{16}\right) + f\left(\frac{7\pi}{16}\right)\right]$

$= \frac{\pi}{8}\left[\sin\left(\frac{\pi}{16}\right) + \sin\left(\frac{3\pi}{16}\right) + \sin\left(\frac{5\pi}{16}\right) + \sin\left(\frac{7\pi}{16}\right)\right]$

≈ 1.006

7.) $\int_{0}^{4} \sqrt{x} \, dx$, $n=4$, $f(x) = \sqrt{x}$:



$h = \frac{4-0}{4} = 1$ so Midpoint Estimate is

$$M_4 = 1 \cdot \left[f(1/2) + f(3/2) + f(5/2) + f(7/2) \right]$$

$$= \sqrt{1/2} + \sqrt{3/2} + \sqrt{5/2} + \sqrt{7/2} \approx 5.384 \; ;$$

<u>Exact</u> : $\int_0^4 \sqrt{x}\, dx = \frac{2}{3} x^{3/2} \Big|_0^4 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2}$

$$= \frac{2}{3} \cdot 8 - 0 = \frac{16}{3} = 5.3333\ldots$$

10.) $\int_{-1}^{0} x^3\, dx$, $n = 5$, $f(x) = x^3$:

$-1 \qquad -4/5 \qquad -3/5 \qquad -2/5 \qquad -1/5 \qquad 0$

$h = \frac{0-(-1)}{5} = \frac{1}{5}$ so Trapezoid Estimate

$$T_5 = \frac{1/5}{2} \left[f(-1) + 2f\left(\tfrac{-4}{5}\right) + 2f\left(\tfrac{-3}{5}\right) + 2f\left(\tfrac{-2}{5}\right) + 2f\left(\tfrac{-1}{5}\right) + f(0) \right]$$

$$= \frac{1}{10} \left[-1 + 2 \cdot \frac{-64}{125} + 2 \cdot \frac{-27}{125} + 2 \cdot \frac{-8}{125} + 2 \cdot \frac{-1}{125} + 0 \right]$$

$$= \frac{1}{10} \left[\frac{-125}{125} + \frac{-128}{125} + \frac{-54}{125} + \frac{-16}{125} + \frac{-2}{125} \right]$$

$$= \frac{-325}{1250} = -0.26$$

15.) $\int_0^2 \sqrt{x}\, dx$, $n = 4$, $f(x) = \sqrt{x}$:

$0 \qquad 1/2 \qquad 1 \qquad 3/2 \qquad 2$

$h = \frac{2-0}{4} = \frac{1}{2}$ so Trapezoid Estimate

$$T_4 = \frac{\frac{1}{2}}{2}\left[f(0) + 2f\left(\tfrac{1}{2}\right) + 2f(1) + 2f\left(\tfrac{3}{2}\right) + f(2)\right]$$

$$= \frac{1}{4}\cdot\left[0 + 2\sqrt{\tfrac{1}{2}} + 2(1) + 2\sqrt{\tfrac{3}{2}} + \sqrt{2}\right] \approx 1.8195;$$

Exact : $\int_0^2 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_0^2 = \frac{2}{3}(2)^{3/2}$

≈ 1.8856

16.) $\int_1^2 \frac{1}{x}\,dx$, $n=5$, $f(x) = \frac{1}{x}$:

$h = \frac{2-1}{5} = \frac{1}{5}$ so Trapezoid Estimate

$$T_5 = \frac{1/5}{2}\left[f(1) + 2f\left(\tfrac{6}{5}\right) + 2f\left(\tfrac{7}{5}\right) + 2f\left(\tfrac{8}{5}\right) + 2f\left(\tfrac{9}{5}\right) + f(2)\right]$$

$$= \frac{1}{10}\left[1 + 2\left(\tfrac{5}{6}\right) + 2\left(\tfrac{5}{7}\right) + 2\left(\tfrac{5}{8}\right) + 2\left(\tfrac{5}{9}\right) + \tfrac{1}{2}\right]$$

$$= \frac{1}{10}\left[1 + \frac{10}{6} + \frac{10}{7} + \frac{10}{8} + \frac{10}{9} + \frac{1}{2}\right] \approx 0.6956;$$

Exact : $\int_1^2 \frac{1}{x}\,dx = \ln|x|\Big|_1^2 = \ln 2 - \cancel{\ln 1}^{\,0}$

≈ 0.6931

18.) $\int_1^2 \frac{1}{x}\,dx$, $f(x) = \frac{1}{x}$, want $|E_n| \le 0.001$:

$f'(x) = \frac{-1}{x^2}$ so $\quad f''(x) = \frac{2}{x^3}$, then

$$\max_{1\le x\le 2} |f''(x)| = \max_{1\le x\le 2} \left|\frac{2}{x^3}\right| = \frac{2}{(1)^3} = 2 \ ;$$

$h = \frac{2-1}{n} = \frac{1}{n}$ so that

$$|E_n| \le (2-1)\ \frac{(\frac{1}{n})^2}{24} \left\{ \max_{1\le x\le 2} |f''(x)| \right\}$$

$$= \frac{1}{24n^2} \cdot \{2\} = \frac{1}{12n^2} \le 0.001 \longrightarrow$$

$$n^2 \ge \frac{1}{12(0.001)} \longrightarrow n \ge \sqrt{\frac{1}{12(0.001)}} \approx 9.13$$

so choose $n = 10$ (or bigger).

20.) $\int_2^8 \frac{1}{\ln t}\,dt$, $f(t) = \frac{1}{\ln t} = (\ln t)^{-1}$,

want $|E_n| \le 0.001$: $\xrightarrow{D}$

$$f'(t) = -(\ln t)^{-2} \cdot \frac{1}{t} = \frac{-(\ln t)^{-2}}{t} \xrightarrow{D}$$

$$f''(t) = \frac{t \cdot 2(\ln t)^{-3} \cdot \frac{1}{t} - -(\ln t)^{-2}(1)}{t^2}$$

$$= \left[\frac{2}{(\ln t)^3} + \frac{1}{(\ln t)^2}\right] \cdot \frac{1}{t^2} = \frac{2+\ln t}{(\ln t)^3 t^2}, \text{ then}$$

$$\max_{2\le t\le 8} |f''(t)| = \max_{2\le t\le 8} \left|\frac{2+\ln t}{(\ln t)^3 t^2}\right| \le \frac{2+\ln 8}{(\ln 2)^3 2^2}$$

$$= \frac{2+\ln 8}{4(\ln 2)^3} \ ; \ h = \frac{8-2}{n} = \frac{6}{n} \longrightarrow$$

$$|E_n| \le (8-2)\,\frac{\left(\frac{6}{n}\right)^2}{24}\left\{\max_{2\le t\le 8}|f''(t)|\right\}$$

$$= \frac{6(36)}{24}\cdot\frac{1}{n^2}\cdot\left\{\frac{2+\ln 8}{4(\ln 2)^3}\right\}$$

$$= \boxed{(9)\cdot\frac{1}{n^2}\cdot\left\{\frac{2+\ln 8}{4(\ln 2)^3}\right\} \le 0.001}$$

$$\rightarrow \quad n^2 \ge \frac{9}{0.001}\cdot\left\{\frac{2+\ln 8}{4(\ln 2)^3}\right\}$$

$$\rightarrow \quad n \ge \sqrt{\frac{9}{0.001}\cdot\left\{\frac{2+\ln 8}{4(\ln 2)^3}\right\}} \approx 166.02$$

so choose $\boxed{n=167}$ (or bigger)

23.) $\int_1^2 \frac{e^t}{t}\,dt$, $f(t)=\frac{e^t}{t}$, want

$|E_n| \le 0.0001$; $\xrightarrow{D} f'(t)=\dfrac{te^t-e^t(1)}{t^2} \xrightarrow{D}$

$$f''(t) = \frac{t^2(te^t+\cancel{e^t}-\cancel{e^t}) - (te^t-e^t)\cdot 2t}{(t^2)^2}$$

$$= \frac{t^3e^t-2t^2e^t+2te^t}{t^4}$$

$$= \frac{te^t(t^2-2t+2)}{t^4} = \frac{e^t(t^2-2t+2)}{t^3}, \text{ then}$$

$$\max_{1\le t\le 2}|f''(t)| = \max_{1\le t\le 2}\left|\frac{e^t\cdot(t^2-2t+2)}{t^3}\right|$$

$$\le \frac{e^2\cdot(2^2-2(2)+2)}{1^3} = 2e^2 \text{ ; and}$$

$h=\frac{2-1}{n}=\frac{1}{n}$ so that

$$|E_n| \le (2-1)\frac{\left(\frac{1}{n}\right)^2}{12}\cdot\left\{\max_{1\le t\le 2}|f''(t)|\right\}$$

$$\le \frac{1}{\cancel{12}_6 n^2}\cdot\{\cancel{2}e^2\} \le \frac{1}{6n^2}\{8\} = \frac{4}{3n^2} \le 0.0001$$

$$\rightarrow n^2 \ge \frac{4}{3(0.0001)} \rightarrow n \ge \sqrt{\frac{4}{0.0003}} \approx 115.5$$

so choose $\boxed{n=116}$ (or bigger)