Worksheet 4

1.) a.) $\mathcal{L}\{5\} = \mathcal{L}\{5 \cdot 1\}$

$= 5\mathcal{L}\{1\} = 5 \cdot \frac{1}{s}$

b.) $\mathcal{L}\{3t-2\} = 3\mathcal{L}\{t\} - 2\mathcal{L}\{1\}$

$= 3 \cdot \frac{1}{s^2} - 2 \cdot \frac{1}{s}$

c.) $\mathcal{L}\{t^2+2t-5\}$

$= \mathcal{L}\{t^2\} + 2\mathcal{L}\{t\} - 5\mathcal{L}\{1\}$

$= \frac{2}{s^3} + 2 \cdot \frac{1}{s^2} - 5 \cdot \frac{1}{s}$

d.) $\mathcal{L}\{t^3 \cdot e^{2t}\} = \frac{3!}{(s-2)^4}$

e.) $\mathcal{L}\{e^t - te^{-t}\}$

$= \mathcal{L}\{e^t\} - \mathcal{L}\{te^{-t}\}$

$= \frac{1}{s-1} - \frac{1}{(s+1)^2}$

f.) $\mathcal{L}\{\sin t + \cos 3t\}$

$= \mathcal{L}\{\sin t\} + \mathcal{L}\{\cos 3t\}$

$= \frac{1}{s^2+1^2} + \frac{s}{s^2+3^2}$

g.) $\mathcal{L}\{e^{-2t}\sin t\} = \dfrac{1}{(s+2)^2+1^2}$

h.) $\mathcal{L}\{e^{t}\cos 4t\} = \dfrac{s-1}{(s-1)^2+4^2}$

2.) a.) $\mathcal{L}^{-1}\left\{\dfrac{3}{s}\right\} = \mathcal{L}^{-1}\left\{3 \cdot \dfrac{1}{s}\right\} = 3 \cdot 1 = 3$

b.) $\mathcal{L}^{-1}\left\{\dfrac{3!}{s^4}\right\} = t^3$

c.) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^5}\right\} = \mathcal{L}^{-1}\left\{\dfrac{1}{4!} \cdot \dfrac{4!}{s^5}\right\} = \dfrac{1}{24} t^4$

d.) $\mathcal{L}^{-1}\left\{\dfrac{1}{s+4}\right\} = \mathcal{L}^{-1}\left\{\dfrac{1}{s-(-4)}\right\} = e^{-4t}$

e.) $\mathcal{L}^{-1}\left\{\dfrac{1}{(s+4)^3}\right\} = \mathcal{L}^{-1}\left\{\dfrac{1}{(s-(-4))^3}\right\}$

$= \mathcal{L}^{-1}\left\{\dfrac{1}{2} \cdot \dfrac{2}{(s-(-4))^3}\right\} = \dfrac{1}{2} \cdot t^2 e^{-4t}$

f.) $\mathcal{L}^{-1}\left\{\dfrac{1}{s} + \dfrac{1}{s^2} + \dfrac{1}{s^3}\right\}$

$= \mathcal{L}^{-1}\left\{\dfrac{1}{s} + \dfrac{1}{s^2} + \dfrac{1}{2} \cdot \dfrac{2}{s^3}\right\} = 1 + t + \dfrac{1}{2} t^2$

g.) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+1}\right\} = \mathcal{L}^{-1}\left\{\dfrac{1}{s^2+1^2}\right\} = \sin t$

h.) $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+1}\right\} = \mathcal{L}^{-1}\left\{\dfrac{s}{s^2+1^2}\right\} = \cos t$

i.) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+3^2}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{3}\cdot\frac{3}{s^2+3^2}\right\} = \frac{1}{3}\cdot \sin 3t$

j.) $\mathcal{L}^{-1}\left\{\frac{s}{s^2+16}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+4^2}\right\} = \cos 4t$

k.) $\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2+9}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{3}\cdot\frac{3}{(s-2)^2+3^2}\right\}$

$= \frac{1}{3}\cdot e^{2t}\sin 3t$

l.) $\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{s-(-1)}{(s-(-1))^2+2^2}\right\}$

$= e^{-t}\cos 2t$

m.) $\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{s+1-1}{(s+1)^2+2^2}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+2^2} + \frac{-1}{(s+1)^2+2^2}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s-(-1)}{(s-(-1))^2+2^2} - \frac{1}{2}\cdot\frac{2}{(s+1)^2+2^2}\right\}$

$= e^{-t}\cos 2t - \frac{1}{2}e^{-t}\sin 2t$

n.) $\frac{2}{s^2-4} = \frac{2}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2} \longrightarrow$

$A(s+2) + B(s-2) = 2$:

Let $s=2$: $4A = 2 \rightarrow A = 1/2$

Let $s=-2$: $-4B = 2 \rightarrow B = -1/2$; then

$$\mathcal{L}^{-1}\left\{\frac{2}{s^2-4}\right\} = \mathcal{L}^{-1}\left\{\frac{1/2}{s-2} + \frac{-1/2}{s+2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{2}\cdot\frac{1}{s-2} - \frac{1}{2}\cdot\frac{1}{s-(-2)}\right\}$$

$$= \frac{1}{2}e^{2t} - \frac{1}{2}e^{-2t}$$

o.) $$\frac{s}{s^2-4s-5} = \frac{s}{(s-5)(s+1)} = \frac{A}{s-5} + \frac{B}{s+1} \longrightarrow$$

$A(s+1) + B(s-5) = s$:

Let $s=5$: $6A = 5 \rightarrow A = 5/6$

Let $s=-1$: $-6B = -1 \rightarrow B = 1/6$; then

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2-4s-5}\right\} = \mathcal{L}^{-1}\left\{\frac{5/6}{s-5} + \frac{1/6}{s+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{5}{6}\cdot\frac{1}{s-5} + \frac{1}{6}\cdot\frac{1}{s+1}\right\}$$

$$= \frac{5}{6}\cdot e^{5t} + \frac{1}{6}\cdot e^{-t}$$

p.) $$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1} \longrightarrow$$

$A(s-1) + Bs = 1$:

Let $s=1$: $B = 1$

Let $s=0$: $-A = 1 \rightarrow A = -1$; then

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} = \mathcal{L}^{-1}\left\{\frac{-1}{s} + \frac{1}{s-1}\right\}$$

$$= -1 + e^{t}$$

q.) $$\frac{s+3}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} \longrightarrow$$

$$As(s+1) + B(s+1) + Cs^2 = s+3 :$$

Let $s=0$: $B = 3$,

Let $s=-1$: $C = 2$,

Let $s=1$: $2A + 2(3) + 2(1) = 4 \longrightarrow$

$2A = -4 \longrightarrow A = -2$; then

$$\mathcal{L}^{-1}\left\{\frac{s+3}{s^2(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{-2}{s} + \frac{3}{s^2} + \frac{2}{s+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{-2\cdot\frac{1}{s} + 3\cdot\frac{1}{s^2} + 2\frac{1}{s+1}\right\}$$

$$= -2\cdot(1) + 3t + 2e^{-t}$$

r.) $$\frac{s^2+1}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4} \longrightarrow$$

$$A(s^2+4) + (Bs+C)s = s^2+1 :$$

Let $s=0$: $4A = 1 \longrightarrow A = 1/4$,

Let $s=2i$: $A(0) + (2Bi+C)2i = (2i)^2+1$

$\rightarrow -4B + 2Ci = -4+1$

$\rightarrow (-4B) + (2C)i = (-3) + (0)i$

$\rightarrow -4B = -3 \rightarrow B = 3/4$ and

$2C = 0 \rightarrow C = 0$; then

$$\mathcal{L}^{-1}\left\{\frac{s^2+1}{s(s^2+4)}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{1}{4}}{s} + \frac{\frac{3}{4}s}{s^2+4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{4}\cdot\frac{1}{s} + \frac{3}{4}\cdot\frac{s}{s^2+2^2}\right\}$$

$$= \frac{1}{4}(1) + \frac{3}{4}\cos 2t$$

5.) $\dfrac{1}{s^2-2s+2} = \dfrac{1}{(s^2-2s+1)+1} = \dfrac{1}{(s-1)^2+1^2}$

then

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2-2s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+1^2}\right\}$$

$$= e^{t}\sin t$$