Section 16.6

1.) $f(x,y) = x^2 + y^2 - 2x \xrightarrow{D}$

$f_x = 2x - 2 = 0 \longrightarrow x = 1$,

$f_y = 2y = 0 \longrightarrow y = 0$, so crit. pt. is $\boxed{(1,0)}$:

$f_{xx} = 2$, $f_{yy} = 2$, $f_{xy} = 0$; so

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = (2)(2) - (0)^2 > 0$

and $f_{xx} = 2 > 0$ (U) so $\boxed{(1,0)}$ determines a min. value of $f(1,0) = -1$.

3.) $f(x,y) = x^2 y - 4x^2 - 4y \xrightarrow{D}$

$f_x = 2xy - 8x = 2x(y-4) = 0 \longrightarrow x = 0$ or $y = 4$,

$f_y = x^2 - 4 = (x-2)(x+2) = 0 \longrightarrow x = 2$ or $x = -2$;

so critical pts. are $\boxed{(2,4)}$ and $\boxed{(-2,4)}$:

$f_{xx} = 2y - 8$, $f_{yy} = 0$, $f_{xy} = 2x$; so

For $(2,4)$: $D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = (0)(0) - (4)^2 = -16 < 0$

so $(2,4)$ determines a saddle point;

For $(-2,4)$: $D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = (0)(0) - (-4)^2 = -16 < 0$

so $(-2,4)$ determines a saddle point.

4.) $f(x,y) = xy - 2y^2 \xrightarrow{D}$

$f_x = y = 0 \longrightarrow y = 0$,

$f_y = x - 4y = 0 \longrightarrow x = 4y$; so

$\boxed{(0,0)}$ is critical pt. :

$f_{XX} = 0$, $f_{YY} = -4$, $f_{XY} = 1$; then

$D = f_{XX} \cdot f_{YY} - (f_{XY})^2 = (0)(-4) - (1)^2 = -1 < 0$,

so $(0,0)$ determines a <u>saddle</u> pt.

6.) $f(X,Y) = X - X^2 + XY \xrightarrow{D}$

$f_X = 1 - 2X + Y = 0 \longrightarrow Y = 2X - 1$,

$f_Y = X = 0$, so critical pt. is

$\boxed{(0,-1)}$:

$f_{XX} = -2$, $f_{YY} = 0$, $f_{XY} = 1$ then

$D = f_{XX} \cdot f_{YY} - (f_{XY})^2 = (-2)(0) - (1)^2 = -1 < 0$

so $(0,-1)$ determines a <u>saddle</u> pt.

7.) $f(X,Y) = e^{-X^2 - Y^2} \xrightarrow{D}$

$f_X = -2X e^{-X^2 - Y^2} = 0 \longrightarrow X = 0$,

$f_Y = -2Y e^{-X^2 - Y^2} = 0 \longrightarrow Y = 0$, so

$\boxed{(0,0)}$ is critical pt. :

$f_{XX} = -2X e^{-X^2 - Y^2} \cdot (-2X) + (-2) e^{-X^2 - Y^2}$,

$f_{YY} = -2Y e^{-X^2 - Y^2} \cdot (-2Y) + (-2) e^{-X^2 - Y^2}$,

$f_{XY} = -2X e^{-X^2 - Y^2} \cdot (-2Y)$; then

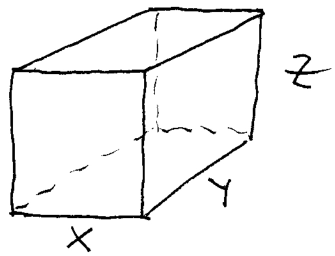
$D = f_{XX} \cdot f_{YY} - (f_{XY})^2 = (-2)(-2) - (0)^2 = 4 > 0$ and

$f_{xx} = -2 < 0$ $(\cap)$, so $(0,0)$ determines a <u>maximum</u> <u>value</u> of
$f(0,0) = e^0 = 1$.

28.) assume $X+Y+Z=60$, maximize $P = XYZ$; and $Z = 60 - X - Y$ so
$P = XY(60 - X - Y) = 60XY - X^2Y - XY^2 \rightarrow$
$\boxed{P = 60XY - X^2Y - XY^2}$; $\xrightarrow{D}$

$P_X = 60Y - 2XY - Y^2 = Y(60 - 2X - Y) = 0$
$\rightarrow$ $Y = 0$ or $60 - 2X - Y = 0$
$\rightarrow$ $\underline{Y = 0}$ or $\underline{Y = 60 - 2X}$;
$P_Y = 60X - X^2 - 2XY = X(60 - X - 2Y) = 0$
$\rightarrow$ $X = 0$ or $60 - X - 2Y = 0$
$\rightarrow$ $\underline{X = 0}$ or $\underline{X = 60 - 2Y}$; so
$X = 0, Y = 0$ or $X = 0, Y = 60 - 2(0)$ or
$Y = 0, X = 60 - 2(0)$ or $Y = 60 - 2X$,
$X = 60 - 2Y \rightarrow X = 60 - 2(60 - 2X) = 60 - 120 + 4X$
$\rightarrow 3X = 60 \rightarrow X = 20, Y = 20$; ~~then~~
critical points are
$(0,0)$, $(0,60)$, $(60,0)$, $(20,20)$:
$X = 0, Y = 0, Z = 60 \rightarrow P = 0$,
$X = 0, Y = 60, Z = 0 \rightarrow P = 0$,
$X = 60, Y = 0, Z = 0 \rightarrow P = 0$,
$X = 20, Y = 20, Z = 20 \rightarrow \boxed{P = 8000}$ MAX

29.)



Given surface area

$2XY + 2XZ + 2YZ = 48 \text{ m}^2$

$\rightarrow XY + XZ + YZ = 24$

$\rightarrow (X+Y)Z = 24 - XY \rightarrow \boxed{Z = \dfrac{24 - XY}{X+Y}}$;

maximize volume

$V = XYZ = XY \cdot \dfrac{24 - XY}{X+Y} = \dfrac{24XY - X^2Y^2}{X+Y} \rightarrow$

$\boxed{V = \dfrac{24XY - X^2Y^2}{X+Y}}$;

$$V_X = \frac{(X+Y)(24Y - 2XY^2) - (24XY - X^2Y^2)(1)}{(X+Y)^2}$$

$$= \frac{\cancel{24XY} + 24Y^2 - 2X^2Y^2 - 2XY^3 - \cancel{24XY} + X^2Y^2}{(X+Y)^2}$$

$$= \frac{24Y^2 - X^2Y^2 - 2XY^3}{(X+Y)^2}$$

$$= \frac{Y^2(24 - X^2 - 2XY)}{(X+Y)^2} = 0 \rightarrow$$

$Y^2 = 0$ (NO) or $24 - X^2 - 2XY = 0$

$\rightarrow \boxed{24 - X^2 = 2XY}$; and

$$V_Y = \frac{(X+Y)(24X - 2X^2Y) - (24XY - X^2Y^2)(1)}{(X+Y)^2}$$

$$= \frac{24X^2 + \cancel{24XY} - 2X^3Y - 2X^2Y^2 - \cancel{24XY} + X^2Y^2}{(X+Y)^2}$$

$$= \frac{24X^2 - 2X^3Y - X^2Y^2}{(X+Y)^2}$$

$$= \frac{X^2(24 - 2XY - Y^2)}{(X+Y)^2} = 0 \longrightarrow$$

$X^2 = 0$ (No) or $24 - 2XY - Y^2 = 0 \rightarrow$

$\boxed{24 - Y^2 = 2XY}$; combining the boxed equations gives

$24 - X^2 = 2XY = 24 - Y^2 \longrightarrow$

$24 - X^2 = 24 - Y^2 \longrightarrow$

$X^2 = Y^2 \longrightarrow \boxed{X = Y}$; then (SUB)

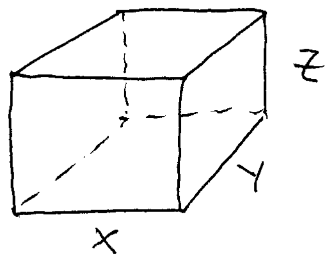
$24 - Y^2 = 2(Y)Y \rightarrow 24 = 3Y^2 \rightarrow$

$Y^2 = 8 \longrightarrow$

$Y = \sqrt{8}$, $X = \sqrt{8}$, $Z = \sqrt{8}$

and max. volume

$V = 8\sqrt{8}$

31.) 

Given volume

$XYZ = 216\ m^3 \rightarrow$

$\boxed{Z = \frac{216}{XY}}$;

minimize surface area

$S = 2XY + 2XZ + 2YZ$

$= 2XY + (2X + 2Y)Z$

$= 2XY + (2X+2Y) \cdot \frac{216}{XY}$

$= 2XY + \frac{432}{Y} + \frac{432}{X} \longrightarrow$

$\boxed{S = 2XY + \frac{432}{Y} + \frac{432}{X}}$; ~~then~~

$S_X = 2Y - \frac{432}{X^2} = 0 \rightarrow \boxed{Y = \frac{216}{X^2}}$;

$S_Y = 2X - \frac{432}{Y^2} = 0 \rightarrow \boxed{X = \frac{216}{Y^2}}$; so

$X = \frac{216}{Y^2} = \frac{216}{\left(\frac{216}{X^2}\right)^2} = \cancel{216} \cdot \frac{X^4}{216^{\cancel{2}}} \longrightarrow$

$216X = X^4 \longrightarrow X^4 - 216X = 0 \rightarrow$

$X(X^3 - 216) = 0 \rightarrow X = 0$ (NO) or

$X = 6$ m., $Y = 6$ m., $Z = 6$ m.

and min. surface area

$S = 216$ m.^{2}