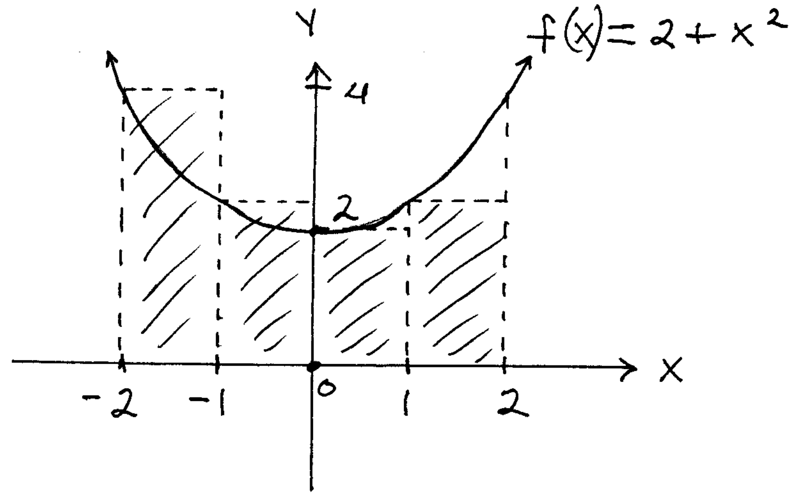
Section 6.1

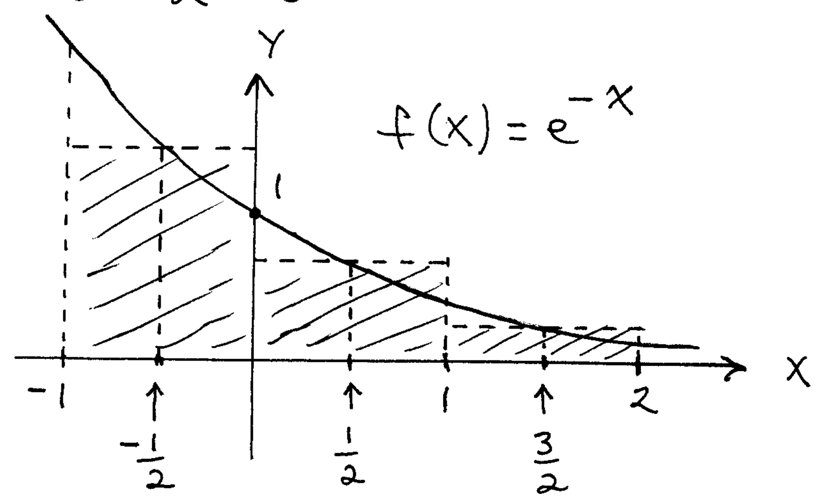
35.)



$$\int_{-2}^{2} (2+x^2)\,dx \approx f(-2)\cdot(1) + f(-1)\cdot(1) + f(0)\cdot(1) + f(-1)\cdot(1)$$

$$= 6 + 3 + 2 + 3 = 14$$

36.)

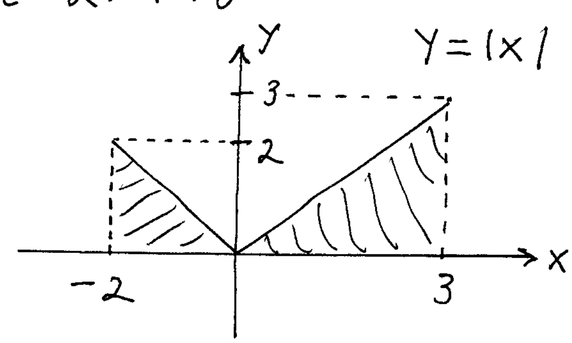


$$\int_{-1}^{2} e^{-x}\,dx \approx f(-1/2)\cdot(1) + f\left(\tfrac{1}{2}\right)\cdot(1) + f\left(\tfrac{3}{2}\right)\cdot(1)$$

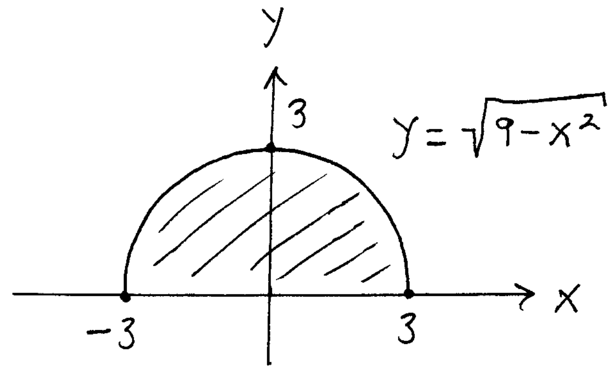
$$= e^{1/2} + e^{-1/2} + e^{-3/2} \approx 2.478$$

61.) $\int_{-2}^{3} |x|\,dx$

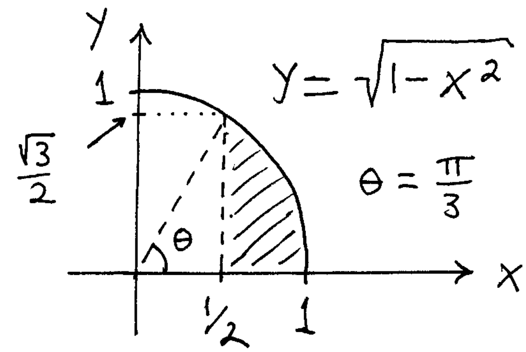
$= \frac{1}{2}(2)(2) + \frac{1}{2}(3)(3) = \frac{13}{2}$



62.) $\int_{-3}^{3} \sqrt{9-x^2}\, dx$



$= \frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi$

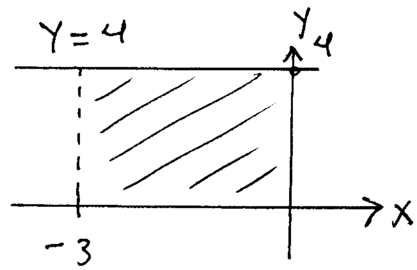


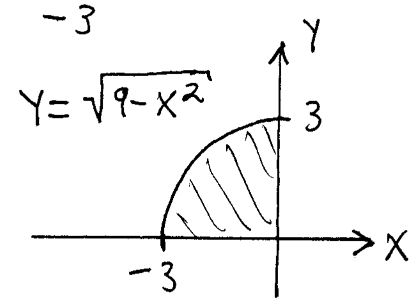
64.) $\int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\, dx$

= Area ◬ − Area ◬

$= \frac{\frac{\pi}{3}}{2\pi}\pi(1)^2 - \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{6}\pi - \frac{\sqrt{3}}{8}$

67.) $\int_{-3}^{0} \left(4-\sqrt{9-x^2}\right) dx = \int_{-3}^{0} 4\, dx - \int_{-3}^{0} \sqrt{9-x^2}\, dx$

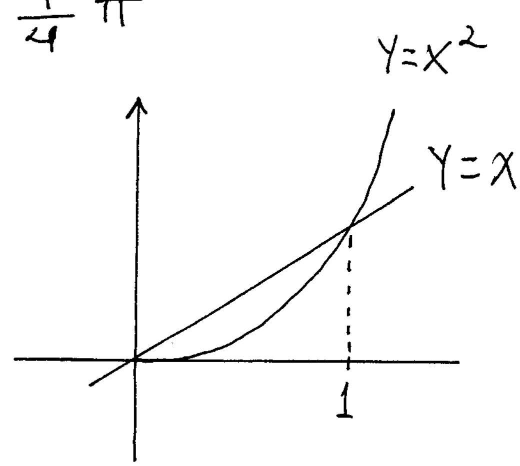


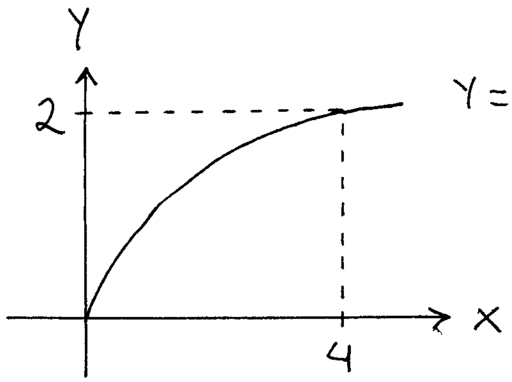


$= (4)(3) - \frac{1}{4}\pi(3)^2 = 12 - \frac{9}{4}\pi$

75.) $x \geq x^2$ for $0 \leq x \leq 1$

so $\int_0^1 x\, dx \geq \int_0^1 x^2\, dx$

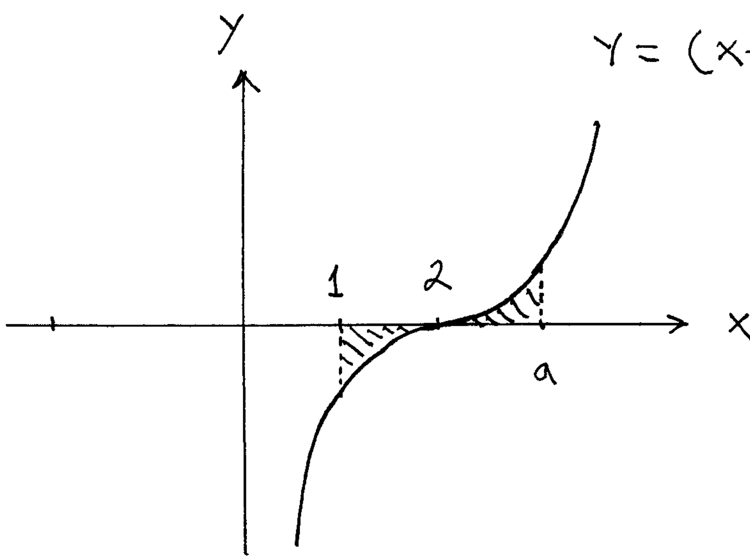


77.) 

$0 \le \sqrt{x} \le 2$
for $0 \le x \le 4$
so that

$$\int_0^4 0\,dx \le \int_0^4 \sqrt{x}\,dx \le \int_0^4 2\,dx \rightarrow$$

$$0 \le \int_0^4 \sqrt{x}\,dx \le 4(2) = 8$$

83.) 

If $\int_1^a (x-2)^3\,dx = 0$, then $a = 3$
(by symmetry)

Math 17B
Kouba
Worksheet 1

1.) Use the limit definition of definite integral to evaluate each of the following integrals. Use n equal subdivisions so that $\Delta x_i = \dfrac{b-a}{n}$ for $i = 1, 2, 3, 4, \cdots, n$. Use right-hand endpoints for sampling points so that the sampling points are $x_i = a + \dfrac{b-a}{n} \cdot i$ for $i = 1, 2, 3, 4, \cdots, n$.

a.) $\displaystyle\int_{-1}^{2} 5 \ dx$

b.) $\displaystyle\int_{0}^{2} (x+3) \ dx$

c.) $\displaystyle\int_{-3}^{0} (x^2 + 2x) \ dx$

d.) $\displaystyle\int_{0}^{1} x^3 \ dx$

e.) $\displaystyle\int_{0}^{1} 2^x \ dx$

HINT 1 : $1 + r + r^2 + r^3 + \cdots + r^m = \dfrac{1 - r^{m+1}}{1 - r}$.

HINT 2 : At some point you will need L'Hopital's Rule.

worksheet 1

1.) a.) $-1 \quad x_1 \quad x_2 \quad x_3 \ \dots \ x_i \ \dots \ 2 = x_n$

$$\Delta x_i = \frac{2-(-1)}{n} = \frac{3}{n} \text{ for } i = 1, 2, 3, \dots$$

and $x_i = -1 + \frac{3}{n} i$; $f(x) = 5$; then

$$\int_{-1}^{2} 5\, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x_i$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} 5 \cdot \frac{3}{n} = \lim_{n \to \infty} \frac{15}{n} \left(\sum_{i=1}^{n} 1 \right)$$

$$= \frac{15}{n} \cdot n = 15$$

b.) $0 \quad x_1 \quad x_2 \quad x_3 \ \dots \ x_i \ \dots \ 2 = x_n$

$$\Delta x_i = \frac{2-0}{n} = \frac{2}{n} \text{ for } i = 1, 2, 3, \dots$$

and $x_i = 0 + \frac{2}{n} i = \frac{2}{n} i$ for $i = 1, 2, 3, \dots$;
$f(x) = x + 3$; then

$$\int_{0}^{2} (x+3)\, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x_i$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} f\left(\frac{2}{n} i\right) \cdot \frac{2}{n} = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{2}{n} i + 3\right) \frac{2}{n}$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{4}{n^2} i + \frac{6}{n}\right) = \lim_{n \to \infty} \left[\frac{4}{n^2} \left(\sum_{i=1}^{n} i \right) + \frac{6}{n} \cdot \left(\sum_{i=1}^{n} 1 \right) \right]$$

$$= \lim_{n \to \infty} \left[\frac{4}{n^2} \cdot \frac{n(n+1)}{2} + \frac{6}{n} \cdot n \right]$$

$$= \lim_{n \to \infty} \left[2 \cdot \left(1 + \frac{1}{n}\right) + 6 \right] = 2(1) + 6 = 8$$

c.) $-3 \quad x_1 \quad x_2 \quad x_3 \ \dots \ x_i \ \dots \ 0 = x_n$

$$\Delta x_i = \frac{0-(-3)}{n} = \frac{3}{n} \text{ for } i = 1, 2, 3, \dots$$

and $x_i = -3 + \frac{3}{n} i$ for $i = 1, 2, 3, \ldots$;

$f(x) = x^2 + 2x$; then

$$\int_{-3}^{0} (x^2 + 2x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} (x_i^2 + 2x_i) \cdot \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-3 + \frac{3}{n} i\right)^2 + 2\left(-3 + \frac{3}{n} i\right) \right] \cdot \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[9 - \frac{18}{n} i + \frac{9}{n^2} i^2 - 6 + \frac{6}{n} i \right] \cdot \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[3 - \frac{12}{n} i + \frac{9}{n^2} i^2 \right] \cdot \frac{3}{n}$$

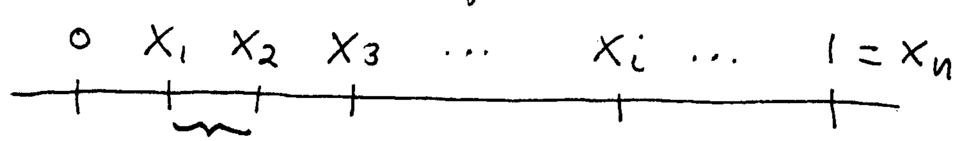
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{9}{n} - \frac{36}{n^2} \cdot i + \frac{27}{n^3} \cdot i^2 \right]$$

$$= \lim_{n\to\infty} \left\{ \frac{9}{n} \cdot \left(\sum_{i=1}^{n} 1 \right) - \frac{36}{n^2} \left(\sum_{i=1}^{n} i \right) + \frac{27}{n^3} \cdot \left(\sum_{i=1}^{n} i^2 \right) \right\}$$

$$= \lim_{n\to\infty} \left\{ \frac{9}{n} \cdot (n) - \frac{36}{\cancel{n^2}} \cdot \frac{\cancel{n}(n+1)}{2} + \frac{27}{\cancel{n^3}\,n^2} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6} \right\}$$

$$= \lim_{n\to\infty} \left\{ 9 - 18 \cdot \left(1 + \frac{1}{n}\right) + \frac{9}{2} \left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) \right\}$$

$$= 9 - 18(1) + \frac{9}{2}(1)(2) = 0$$

d.) 

$$\Delta x_i = \frac{1-0}{n} = \frac{1}{n} \text{ for } i = 1, 2, 3, \ldots$$

and $x_i = 0 + \frac{1}{n} i = \frac{i}{n}$ for $i = 1, 2, 3, \ldots$;

$f(x) = x^3$; then

$$\int_0^1 x^3\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\cdot\frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^3}\cdot\frac{1}{n} = \lim_{n\to\infty} \frac{1}{n^4}\left(\sum_{i=1}^{n} i^3\right)$$

$$= \lim_{n\to\infty} \frac{1}{n^4}\cdot\left(\frac{n(n+1)}{2}\right)^2 = \lim_{n\to\infty} \frac{1}{\cancel{n^4}\,n^2}\,\frac{\cancel{n^2}(n+1)^2}{4}$$

$$= \lim_{n\to\infty} \frac{1}{4}\cdot\left(\frac{n+1}{n}\right)^2 = \lim_{n\to\infty} \frac{1}{4}\left(1+\frac{1}{n}\right)^2 = \frac{1}{4}(1)^2 = \frac{1}{4}$$

e.) 

$$\Delta x_i = \frac{1-0}{n} = \frac{1}{n} \text{ for } i = 1, 2, 3, \ldots$$

and $x_i = 0 + \frac{1}{n} i = \frac{i}{n}$ for $i = 1, 2, 3, \ldots$;

$f(x) = 2^x$; ~~then~~

$$\int_0^1 2^x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\cdot\frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} 2^{i/n}\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty} \left\{\frac{1}{n}\cdot\left(2^{1/n} + 2^{2/n} + 2^{3/n} + \cdots + 2^{n/n}\right)\right\}$$

$$= \lim_{n\to\infty} \left\{\frac{1}{n}\left((2^{1/n}) + (2^{1/n})^2 + (2^{1/n})^3 + \cdots + (2^{1/n})^n\right\}\right.$$

$$= \lim_{n\to\infty} \left\{\frac{1}{n}\cdot 2^{1/n}\cdot\left(1 + (2^{1/n}) + (2^{1/n})^2 + (2^{1/n})^3 + \cdots + (2^{1/n})^{n-1}\right)\right\}$$

$$= \lim_{n\to\infty} \frac{1}{n}\cdot 2^{1/n}\cdot \frac{1-(2^{1/n})^{(n-1)+1}}{1-2^{1/n}}$$

$$= \lim_{n\to\infty} \frac{1}{n}\cdot 2^{1/n}\cdot \frac{1-2}{1-2^{1/n}}$$

$$= \lim_{n\to\infty} \frac{-1}{n} \cdot \frac{2^{1/n}}{1-2^{1/n}} \cdot \frac{2^{-1/n}}{2^{-1/n}}$$

$$= \lim_{n\to\infty} \frac{-1}{n} \cdot \frac{2^0}{2^{-1/n}-2^0}$$

$$= \lim_{n\to\infty} \frac{\frac{-1}{n}}{2^{-1/n}-1}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{n\to\infty} \frac{\cancel{1/n^2}}{2^{-1/n} \cdot \cancel{\frac{1}{n^2}} \cdot \ln 2}$$

$$= \lim_{n\to\infty} \frac{1}{2^{-1/n} \cdot \ln 2}$$

$$= \frac{1}{2^0 \cdot \ln 2}$$

$$= \frac{1}{\ln 2}$$