Math 17B
Kouba
Using Vectors to Determine Equations for Lines (in R^2 and R^3) and Planes (in R^3)

EQUATIONS OF LINES IN R^2 and R^3

I.) Find an equation in parametric form of the line L passing through the point (x_0, y_0) and in the direction of (parallel to) vector $\begin{pmatrix} a \\ b \end{pmatrix}$. Let (x, y) be a random point on line L. Form vectors $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$. Then

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + t \begin{pmatrix} a \\ b \end{pmatrix} \quad \text{(vector equation of line } L\text{) for some } t \in R \longrightarrow$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} at \\ bt \end{pmatrix} \longrightarrow$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 + at \\ y_0 + bt \end{pmatrix} \longrightarrow$$

$$L: \begin{cases} x = x_0 + at \\ y = y_0 + bt \end{cases} \text{ for } t \in R \quad \text{(equation of line in parametric form)}$$

II.) Find an equation in parametric form of the line L passing through the point (x_0, y_0, z_0) and in the direction of (parallel to) vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$. Let (x, y, z) be a random point on line L. Form vectors $\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Then

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + t \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \text{(vector equation of line } L\text{) for some } t \in R \longrightarrow$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \begin{pmatrix} at \\ bt \\ c \end{pmatrix} \longrightarrow$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 + at \\ y_0 + bt \\ z_0 + ct \end{pmatrix} \longrightarrow$$

$$L: \begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \quad \text{for } t \in R \end{cases} \quad \text{(equation of line in parametric form)}$$

EQUATIONS OF PLANES IN R^3

Find an equation of the plane passing through the point (x_0, y_0, z_0) and which is perpendicular to (normal to) the vector $N = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$. Let (x, y, z) be a random point on the plane. Form vector $V = \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}$, which lies in the plane. Then

$$N \perp V \quad \longrightarrow \quad N \bullet V = 0 \quad \longrightarrow$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} = 0 \quad \longrightarrow$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$