Math 17B
Kouba
Logistic Growth Equation

Ex: assume growth rate is

$$\frac{dN}{dt} = N\left(1-\frac{N}{20}\right) \quad \text{with} \quad N(0)=2$$

(Logistic Growth Equation : per capita growth rate is density dependent ; see p. 486)

Solve for N and graph it.

Solution: Equation is separable so

$$\frac{1}{N\left(1-\frac{N}{20}\right)}\,dN = dt \longrightarrow$$

$$\int \frac{1}{N\left(1-\frac{N}{20}\right)} \cdot \frac{20}{20}\,dN = \int 1\,dt \longrightarrow$$

$$\int \frac{20}{N(20-N)}\,dN = t + C_1 \longrightarrow$$

$$\int \left[\frac{A}{N} + \frac{B}{20-N}\right] dN = t + C_1$$

$$\Big(A(20-N) + BN = 20 \quad ;$$

$$\underline{\text{Let } N=20}: \ 20B = 20 \rightarrow B = 1 \, ,$$

$$\underline{\text{Let } N=0}: \ 20A = 20 \rightarrow A = 1 \Big) \longrightarrow$$

$$\int \left[\frac{1}{N} + \frac{1}{20-N}\right] dN = t + c_1 \rightarrow \quad \left(\begin{matrix}\text{assume} \\ N<20.\end{matrix}\right)$$

$$\ln N - \ln(20-N) = t + c_1 \rightarrow$$

$$\ln\left(\frac{N}{20-N}\right) = t + c_1 \text{ ; and } t=0,\ N=2 \rightarrow$$

$$\ln\left(\frac{1}{9}\right) = 0 + c_1 \rightarrow c_1 = \ln\left(\frac{1}{9}\right) \rightarrow$$

$$\ln\left(\frac{N}{20-N}\right) = t + \ln\left(\frac{1}{9}\right) \text{ ; now solve for } N \text{ :}$$

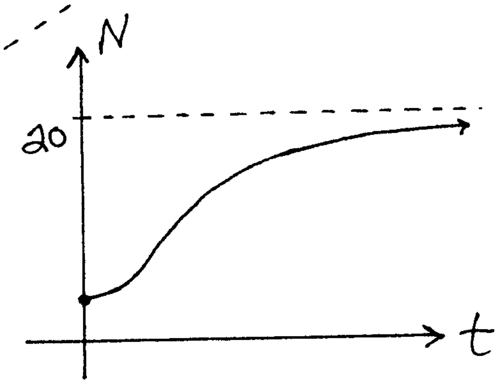
$$e^{\ln\left(\frac{N}{20-N}\right)} = e^{t+\ln\left(\frac{1}{9}\right)} = e^{t} e^{\ln\left(\frac{1}{9}\right)} \rightarrow$$

$$\frac{N}{20-N} = e^{t} \cdot \frac{1}{9} \rightarrow N = \frac{20}{9}e^{t} - \frac{1}{9}e^{t}N \rightarrow$$

$$N + \frac{1}{9}e^{t}N = \frac{20}{9}e^{t} \rightarrow$$

$$\left(1 + \frac{1}{9}e^{t}\right)N = \frac{20}{9}e^{t} \rightarrow$$

$$N = \frac{\frac{20}{9}e^{t}}{1+\frac{1}{9}e^{t}} \cdot \frac{9}{9} \rightarrow$$



$$\boxed{N = \frac{20e^{t}}{9+e^{t}}} \text{ ;}$$

<u>Note</u>: $\lim_{t\to\infty} \frac{20e^{t}}{9+e^{t}} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{t\to\infty} \frac{20\cancel{e^{t}}}{\cancel{e^{t}}} = 20$

(called carrying capacity)