Math 17B (Spring 2017)
Kouba
Exam 1

Your Name : KEY

Your EXAM ID Number ___________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO HAVE ANOTHER STUDENT TAKE YOUR EXAM FOR YOU. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive LITTLE or NO credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 8 pages, including the cover page.

6. You will be graded on proper use of integral, derivative, and limit notation.

7. You have until 10:50 a.m. to finish the exam. Students who fail to stop working when time is called may have points deducted from their exam scores. Thank you for your cooperation.

8. You may use the following trig identities:

a.) $\sin^2\theta + \cos^2\theta = 1$
b.) $1 + \tan^2\theta = \sec^2\theta$
c.) $\cos 2\theta = 2\cos^2\theta - 1$
$\quad = 1 - 2\sin^2\theta$
$\quad = \cos^2\theta - \sin^2\theta$
d.) $\sin 2\theta = 2\sin\theta\cos\theta$

1.) (5 pts. each) Use any method to integrate each of the following. DO NOT SIMPLIFY answers.

a.) $\int (1-x)(x^2+x)\ dx$ $= \int (\cancel{x^2} + x - x^3 - \cancel{x^2})\, dx$

$= \frac{1}{2}x^2 - \frac{1}{4}x^4 + C$

b.) $\int \frac{x^2 - x + 7}{x}\ dx$ $= \int \left(\frac{x^2}{x} - \frac{x}{x} + \frac{7}{x}\right) dx$

$= \int \left(x - 1 + 7 \cdot \frac{1}{x}\right) dx$

$= \frac{1}{2}x^2 - x + 7 \ln|x| + C$

c.) $\int \sec^2 x \tan^2 x\ dx$ (Let $u = \tan x \xrightarrow{D}$ $du = \sec^2 x\, dx$)

$= \int u^2\, du = \frac{1}{3}u^3 + C = \frac{1}{3}(\tan x)^3 + C$

d.) $\int \frac{x+2}{x^2 - 3x - 4}\ dx$ (HINT: Use Partial Fraction.)

$= \int \frac{x+2}{(x-4)(x+1)}\, dx = \int \left[\frac{A}{x-4} + \frac{B}{x+1}\right] dx$

$= \int \left(\frac{6/5}{x-4} + \frac{-1/5}{x+1}\right) dx$

($A(x+1) + B(x-4) = x+2$

Let $x=-1$: $-5B = 1 \rightarrow B = -1/5$

Let $x=4$: $5A = 6 \rightarrow A = 6/5$) $= \frac{6}{5}\ln|x-4| - \frac{1}{5}\ln|x+1| + C$

e.) $\int xe^{-x}\ dx$ (Let $u = x$, $dv = e^{-x}dx \rightarrow$ $du = dx$, $v = -e^{-x}$)

$= -xe^{-x} - \int e^{-x} dx$

$= -xe^{-x} + (-e^{-x}) + C$

f.) $\int \frac{x+2}{1-x}\,dx$ (Let $u = 1-x \xrightarrow{D} du = -dx$ and $x = 1-u$)

$$= -\int \frac{(1-u)+2}{u}\,du = -\int \frac{3-u}{u}\,du = -\int\left(\frac{3}{u}-1\right)du$$

$$= -\left(3\ln|u| - u\right) + C = -3\ln|1-x| + (1-x) + C$$

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g.) $\int \frac{e^x}{4+e^{2x}}\,dx$ $= \int \frac{e^x}{4+(e^x)^2}\,dx = \int \frac{e^x}{2^2+(e^x)^2}\,dx$

(Let $u = e^x \xrightarrow{D} du = e^x\,dx$)

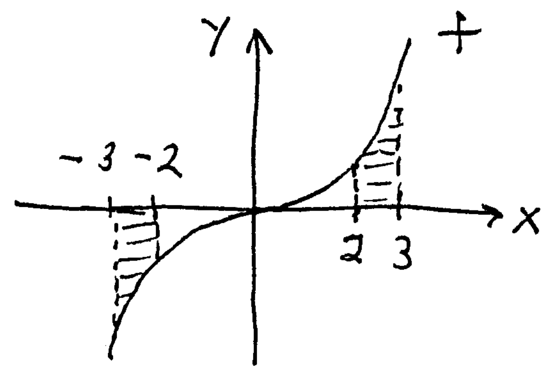
$$= \int \frac{1}{2^2+u^2}\,du = \frac{1}{2}\arctan\frac{u}{2} + C = \frac{1}{2}\arctan\frac{e^x}{2} + C$$

2.) (6 pts.) Assume that $y = f(x)$ is an odd function and $\int_{-2}^{3} f(x)\,dx = 5$. Determine the value of $\int_{-3}^{-2} f(x)\,dx$.

$$5 = \int_{-2}^{3} f(x)\,dx = \underbrace{\int_{-2}^{2} f(x)\,dx}_{0 \text{ since } f \text{ odd}} + \int_{2}^{3} f(x)\,dx$$

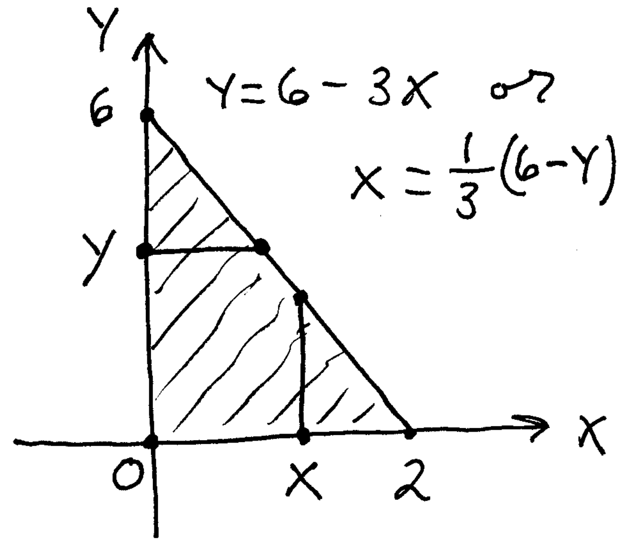
$$\rightarrow \int_{2}^{3} f(x)\,dx = 5$$

$$\rightarrow \int_{-3}^{-2} f(x)\,dx = -5$$



$f(-x) = -f(x)$

4.) (6 pts. each) SET UP BUT DO NOT EVALUATE the definite integral(s) which represents the AREA of the region bounded by the graphs of $y = 6 - 3x$, $y = 0$, and $x = 0$ using



a.) vertical cross-sections.

$$\text{Area} = \int_0^2 (6 - 3x)\, dx$$

b.) horizontal cross-sections.

$$\text{Area} = \int_0^6 \frac{1}{3}(6 - y)\, dy$$

5.) (7 pts.) The temperature T (in degrees Fahrenheit) of a room at time t hours is given by $T = 75 + \cos 2t$. Find the *average* temperature of the room from $t = 0$ to $t = \pi$ hours.

$$\text{AVE} = \frac{1}{\pi - 0} \int_0^{\pi} (75 + \cos 2t)\, dt$$

$$= \frac{1}{\pi}\left(75t + \frac{1}{2}\sin 2t\right)\Big|_0^{\pi}$$

$$= \frac{1}{\pi}\left(75\pi + \frac{1}{2}\cancelto{0}{\sin 2\pi}\right) - \frac{1}{\pi}\left(75(0) + \frac{1}{2}\cancelto{0}{\sin 0}\right)$$

$$= 75\ ^\circ F$$

6.) (8 pts.) Use the limit definition of the definite integral (for convenience, you may choose equal subdivisions and right-hand endpoints) to evaluate $\int_1^2 3x^2\,dx$.

$\boxed{f(x) = 3x^2}$

Divide $[1,2]$ into n equal parts each of length $\boxed{\Delta x_i = \frac{2-1}{n} = \frac{1}{n}}$;

$x_0 = 1 \quad x_1 \quad x_2 \quad x_3 \quad \cdots \quad x_i \quad \cdots \quad 2 = x_n$; ~~then~~

$1+\frac{1}{n} \quad 1+\frac{2}{n} \quad 1+\frac{3}{n} \cdots \quad \boxed{x_i = 1 + \frac{i}{n}}$

$$\int_1^2 3x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 3\left(1+\frac{i}{n}\right)^2 \cdot \frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 3\left(1+\frac{2i}{n}+\frac{i^2}{n^2}\right)\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(3+\frac{6}{n}i+\frac{3}{n^2}i^2\right)\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}+\frac{6}{n^2}i+\frac{3}{n^3}i^2\right)$$

$$= \lim_{n\to\infty} \left[\frac{3}{n}\cdot\left(\sum_{i=1}^{n} 1\right)+\frac{6}{n^2}\left(\sum_{i=1}^{n} i\right)+\frac{3}{n^3}\left(\sum_{i=1}^{n} i^2\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{3}{\cancel{n}}\cdot(\cancel{n})+\frac{6}{n^{\cancel{2}}}\cdot\frac{1}{2}\cancel{n}(n+1)+\frac{\cancel{3}}{n^3}\cdot\frac{1}{\cancel{6}_{2}}n(n+1)(2n+1)\right]$$

$$= \lim_{n\to\infty} \left[3+3(1)\left(1+\frac{1}{n}\right)+\frac{1}{2}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right]$$

$$= 3 + 3(1) + \frac{1}{2}(1)(2) = \boxed{7}$$

7.) (8 pts.) Find the length of the graph of $y = (2/3)x^{3/2}$ on the interval $[0, 3]$.

$$ARC = \int_0^3 \sqrt{1+(Y')^2}\,dx = \int_0^3 \sqrt{1+(\cancel{\tfrac{2}{3}}\cancel{\tfrac{3}{2}}x^{1/2})^2}\,dx$$

$$= \int_0^3 \sqrt{1+x}\,dx$$

$$= \frac{2}{3}(1+x)^{3/2}\Big|_0^3$$

$$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$$

$$= \frac{2}{3}(8) - \frac{2}{3}(1)$$

$$= \frac{14}{3}$$

8.) (8 pts.) Rain is falling in Davis, CA, at the rate of $\dfrac{4}{1+t}$ inches per hour for t hours. Find the total amount of rainfall for $t = 0$ to $t = 3$ hours.

$$TOTAL = \int_0^3 \underbrace{\frac{4}{1+t}}_{\text{in./hr.}}\ \underbrace{dt}_{\text{hrs.}}$$

$$= 4\ln|1+t|\Big|_0^3$$

$$= 4\ln 4 - 4\underbrace{\cancel{\ln 1}}_{0}$$

$$= 4\ln 4$$

$$\approx 5.545 \text{ in.}$$

9.) (8 pts.) Find an equation of the line tangent to the graph of $F(x) = x^2 - \int_1^x (\ln t)^3\,dt$ at $x = 1$. (HINT: Use FTC1.)

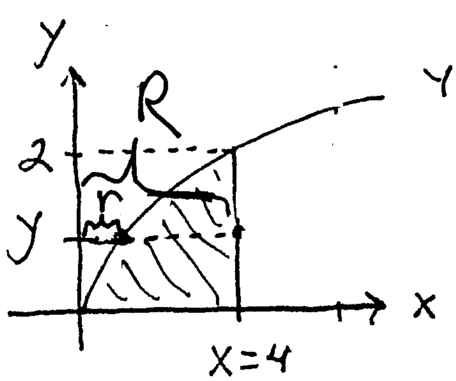
$\xrightarrow{D} F'(x) = 2x - (\ln x)^3 \rightarrow F'(1) = 2(1) - (\overset{0}{\ln 1})^3 = 2$

and $F(1) = (1)^2 - \int_1^1 (\ln t)^3\,dt = 1 - 0 = 1$

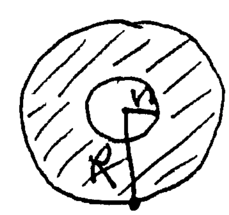
$\rightarrow$ tangent line is

$Y - 1 = 2(X - 1)$ or $Y = 2X - 1$

10.) (8 pts.) Consider the region R bounded by the graphs of $y = \sqrt{x}$, $y = 0$, and $x = 4$. SET UP BUT DO NOT EVALUATE the definite integral representing the VOLUME of the solid formed by revolving R about the y-axis.



$Y = \sqrt{X}$
or $X = Y^2$



Area of slice at y is

$A(y) = \pi R^2 - \pi r^2$

$= \pi(4)^2 - (Y^2)^2$, so

$\text{Volume} = \int_0^2 [\pi(4)^2 - \pi(Y^2)^2]\,dy$

The following EXTRA CREDIT problem is OPTIONAL. It is worth 8 points.

1.) Use any method to integrate : $\displaystyle\int \frac{1}{\sqrt{x+1}-\sqrt{x}}\,dx$

$$= \int \frac{1}{\sqrt{x+1}-\sqrt{x}} \cdot \frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}\, dx$$

$$= \int \frac{\sqrt{x+1}+\sqrt{x}}{(x+1)-x}\, dx$$

$$= \int \left(\sqrt{x+1}-\sqrt{x}\right) dx$$

$$= \frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2} + c$$