to assign a suit to each card and thus a total of 4^3 ways. Combining the different steps, we find that there are

$$13 \cdot \binom{4}{2}\binom{12}{3} \cdot 4^3 = 1{,}098{,}240$$

ways to pick exactly one pair.

Section 12.1 Problems

12.1.1

1. Suppose that you want to investigate the influence of light and fertilizer levels on plant performance. You plan to use five fertilizer and two light levels. For each combination of fertilizer and light level, you want four replicates. What is the total number of replicates?

2. Suppose that you want to investigate the effects of leaf damage on the performance of drought-stressed plants. You plan to use three levels of leaf damage and four different watering protocols. For each combination of leaf damage and watering protocol, you plan to have three replicates. What is the total number of replicates?

3. *Coleomegilla maculata*, a lady beetle, is an important predator of egg masses of *Ostrinia nubialis*, the European corn borer. *C. maculata* also feeds on aphids and maize pollen. To study its food preferences, you choose two satiation levels for *C. maculata* and combinations of two of the three food sources (i.e., either egg masses and aphids, egg masses and pollen, or aphids and pollen). For each experimental protocol, you want 20 replicates. What is the total number of replicates?

4. To test the effects of a new drug, you plan the following clinical trial: Each patient receives the new drug, an established drug, or a placebo. You enroll 50 patients. In how many ways can you assign them to the three treatments?

5. The Muesli-Mix is a popular breakfast hangout near a campus. A typical breakfast there consists of one beverage, one bowl of cereal, and a piece of fruit. If you can choose among three different beverages, seven different cereals, and four different types of fruit, how many choices for breakfast do you have?

6. To study sex differences in food preferences in rats, you offer one of three choices of food to each rat. You plan to have 12 rats for each food-and-sex combination. How many rats will you need?

7. The genome of the HIV virus consists of 9749 nucleotides. There are four different types of nucleotides. Determine the total number of different genomes of size 9749 nucleotides.

8. Automated chemical synthesis of DNA has made it possible to custom-order moderate-length DNA sequences from commercial suppliers. Assume that a single nucleotides weighs about 5.6×10^{-22} gram and that there are four kinds of nucleotides. If you wish to order all possible DNA sequences of a fixed length, at what length will your order exceed **(a)** 100 kg and **(b)** the mass of the Earth (5.9736×10^{24} kg)?

12.1.2

9. You plan a trip to Europe during which you wish to visit London, Paris, Amsterdam, Rome, and Heidelberg. Because you want to buy a railway ticket before you leave, you must decide on the order in which you will visit these five cities. How many different routes are there?

10. Five people line up for a photograph. How many different lineups are possible?

11. You have just bought seven different books. In how many ways can they be arranged on your bookshelf?

12. Four cars arrive simultaneously at an intersection. Only one car can go through at a time. In how many different ways can they leave the intersection?

13. How many four-letter words with no repeated letters can you form from the 26 letters of the alphabet?

14. A committee of 3 people must be chosen from a group of 10. The committee consists of a president, a vice president, and a treasurer. How many committees can be selected?

15. Three different awards are to be given to a class of 15 students. Each student can receive at most one award. Count the number of ways these awards can be given out.

16. You have just enough time to play 4 songs out of 10 from your favorite CD. In how many ways can you program your CD player to play the 4 songs?

17. Six customers arrive at a bank at the same time. Only one customer at a time can be served. In how many ways can the six customers be served?

18. An amino acid is encoded by triplet nucleotides. How many different amino acids are possible if there are four different nucleotides that can be chosen for a triplet?

12.1.3

19. A bag contains 10 different candy bars. You are allowed to choose 3. How many choices do you have?

20. During International Movie Week, 60 movies are shown. You have time to see 5 movies. How many different plans can you make?

21. A committee of 3 people must be formed from a group of 10. How many committees can there be if no specific tasks are assigned to the members?

22. A standard deck contains 52 different cards. In how many ways can you select 5 cards from the deck?

23. An urn contains 15 different balls. In how many ways can you select 4 balls without replacement?

24. Twelve people wait in front of an elevator that has room for only 5. Count the number of ways that the first group of people to take the elevator can be chosen.

25. Four A's and five B's are to be arranged into a nine-letter word. How many different words can you form?

26. Suppose that you want to plant a flower bed with four different plants. You can choose from among eight plants. How may different choices do you have?

27. Amin owns a 4-GB music storage device that holds 1000 songs. How many different playlists of 20 songs are there if the order of the songs is important?

28. A bookstore has 300 science fiction books. Molly wants to buy 5 of the 300 science fiction books. How many selections are there?

12.1.4

29. A box contains five red and four blue balls. You choose two balls.

(a) How many possible selections contain exactly two red balls, how many exactly two blue balls, and how many exactly one of each color?

(b) Show that the sum of the number of choices for the three cases in (a) is equal to the number of ways that you can select two balls out of the nine balls in the box.

30. Twelve children are divided up into three groups, of five, four, and three children, respectively. In how many ways can this be done if the order within each group is not important?

31. Five A's, three B's, and six C's are to be arranged into a 14-letter word. How many different words can you form?

32. A bag contains 45 beans of three different varieties. Each variety is represented 15 times in the bag. You grab 9 beans out of the bag.

(a) Count the number of ways that each variety can be represented exactly three times in your sample.

(b) Count the number of ways that only one variety appears in your sample.

33. Let $S = \{a, b, c\}$. List all possible subsets, and argue that the total number of subsets is $2^3 = 8$.

34. Suppose that a set contains n elements. Argue that the total number of subsets of this set is 2^n.

35. In how many ways can Brian, Hilary, Peter, and Melissa sit on a bench if Peter and Melissa want to be next to each other?

36. Paula, Cindy, Gloria, and Jenny have dinner at a round table. In how many ways can they sit around the table if Cindy wants to sit to the left of Paula?

37. In how many ways can you form a committee of three people from a group of seven if two of the people do not want to serve together?

38. In how many ways can you form two committees of three people each from a group of nine if

(a) no person is allowed to serve on more than one committee?

(b) people can serve on both committees simultaneously?

39. A collection contains seeds for four different annual and three different perennial plants. You plan a garden bed with three different plants, and you want to include at least one perennial. How many different selections can you make?

40. In diploid organisms, chromosomes appear in pairs in the nuclei of all cells except gametes (sperm or ovum). Gametes are formed during meiosis, a process in which the number of chromosomes in the nucleus is halved; that is, only one member of each pair of chromosomes ends up in a gamete. Humans have 23 pairs of chromosomes. How may kinds of gametes can a human produce?

41. Sixty patients are enrolled in a small clinical trial to test the efficacy of a new drug against a placebo and the currently used drug. The patients are divided into 3 groups of 20 each. Each group is assigned one of the three treatments. In how many ways can the patients be assigned?

42. One hundred patients wish to enroll in a small study in which patients are divided into four groups of 25 patients each. In how many ways can this be done if no patient is to be assigned to more than one group?

43. Expand $(x + y)^4$. 44. Expand $(2x - 3y)^5$.

45. In how many ways can four red and five black cards be selected from a standard deck of cards if cards are drawn without replacement?

46. In how many ways can two aces and three kings be selected from a standard deck of cards if cards are drawn without replacement?

47. In the game of poker, determine the number of ways exactly two pairs can be picked.

48. In the game of poker, determine the number of ways a *flush* (five cards of the same suit) can be picked.

49. In the game of poker, determine the number of ways *four of a kind* (four cards of the same value, plus one other cards) can be picked.

50. In the game of poker, determine the number of ways a *straight* (five cards with consecutive values, such as A 2 3 4 5 or 7 8 9 10 J or 10 J Q K A, but not all of the same suit) can be picked.

51. **Counterpoint** *Counterpoint* is a musical term that means the combination of simultaneous voices; it is synonymous with *polyphony*. In *triple counterpoint*, three voices are arranged such that any voice can take any place of the three possible positions: highest, intermediate, and lowest voice. In how many ways can the three voices be arranged?

52. **Counterpoint** *Counterpoint* is a musical term that means the combination of simultaneous voices; it is synonymous with *polyphony*. In *quintuple counterpoint*, five voices are arranged such that any voice can take any place of the five possible positions: from highest to lowest voice. In how many ways can the five voices be arranged?

12.2 What Is Probability?

12.2.1 Basic Definitions

A **random experiment** is a repeatable experiment in which the outcome is uncertain. Tossing a coin and rolling a die are examples of random experiments. The set of all possible outcomes of a random experiment is called the **sample space** and is often denoted by Ω (uppercase Greek omega). We look at some examples in which we describe random experiments and give the associated sample space.

EXAMPLE 1 Suppose that we toss a coin labeled heads (H) on one side and tails (T) on the other. If we toss the coin once, the possible outcomes are H and T, and the sample space is

Section 12.2 Problems

■ 12.2.1

In Problems 1–4, determine the sample space for each random experiment.

1. The random experiment consisting of tossing a coin three times.

2. The random experiment consisting of rolling a six-sided die twice.

3. An urn contains five balls numbered 1–5, respectively. The random experiment consists of selecting two balls simultaneously without replacement.

4. An urn contains six balls numbered 1–6, respectively. The random experiment consists of selecting five balls simultaneously without replacement.

In Problems 5–8, assume that

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$A = \{1, 3, 5\}$, *and* $B = \{1, 2, 3\}$.

5. Find $A \cup B$ and $A \cap B$.

6. Find A^c and show that $(A^c)^c = A$.

7. Find $(A \cup B)^c$.

8. Are A and B disjoint?

In Problems 9–12, assume that

$$\Omega = \{1, 2, 3, 4, 5\}$$

$P(1) = 0.1$, $P(2) = 0.2$, *and* $P(3) = P(4) = 0.05$. *Furthermore, assume that* $A = \{1, 3, 5\}$ *and* $B = \{2, 3, 4\}$.

9. Find $P(5)$.

10. Find $P(A)$ and $P(B)$.

11. Find $P(A^c)$.

12. Find $P(A \cup B)$.

In Problems 13–15, assume that

$$\Omega = \{1, 2, 3, 4\}$$

and $P(1) = 0.1$. *Furthermore, assume that* $A = \{2, 3\}$ *and* $B = \{3, 4\}$, $P(A) = 0.7$, *and* $P(B) = 0.5$.

13. Find $P(3)$.

14. Set $C = \{1, 2\}$. Find $P(C)$.

15. Find $P((A \cap B)^c)$.

16. Assume that $P(A \cap B^c) = 0.1$, $P(B \cap A^c) = 0.5$, and $P((A \cup B)^c) = 0.2$. Find $P(A \cap B)$.

17. Assume that $P(A \cap B) = 0.1$, $P(A) = 0.4$, and $P(A^c \cap B^c) = 0.2$. Find $P(B)$.

18. Assume that $P(A) = 0.4$, $P(B) = 0.4$, and $P(A \cup B) = 0.7$. Find $P(A \cap B)$ and $P(A^c \cap B^c)$.

19. Show the second of the additional properties, namely,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad (12.5)$$

(a) Use a diagram to show that B can be written as a disjoint union of the sets $A \cap B$ and $B \cap A^c$.

(b) Use a diagram to show that $A \cup B$ can be written as a disjoint union of the sets A and $B \cap A^c$.

(c) Use your results in (a) and (b) to show that

$$P(A \cup B) = P(A) + P(B \cap A^c)$$

and

$$P(B \cap A^c) = P(B) - P(A \cap B)$$

Conclude from these two equations that (12.5) holds.

20. If $A \subset B$, we can define the difference between the two sets A and B, denoted by $B - A$ (read "B minus A"),

$$B - A = B \cap A^c$$

as illustrated in Figure 12.12.

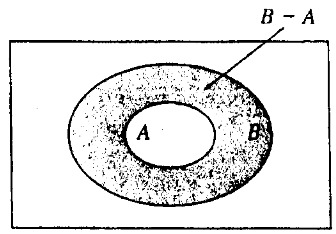

Figure 12.12 The set A is contained in B. The shaded area is the difference of A and B, $B - A$.

Go through the following steps to show that the difference rule

$$P(B - A) = P(B) - P(A) \qquad (12.6)$$

holds:

(a) Use the diagram in Figure 12.12 to show that B can be written as a disjoint union of A and $B - A$.

(b) Use your result in (a) to conclude that

$$P(B) = P(A) + P(B - A)$$

and show that (12.6) follows from this equation.

(c) An immediate consequence of (12.6) is the result that if $A \subset B$, then

$$P(A) \leq P(B)$$

Use (12.6) to show this inequality.

■ 12.2.2

21. Toss two fair coins and find the probability of at least one head.

22. Toss three fair coins and find the probability of no heads.

23. Toss four fair coins and find the probability of exactly two heads.

24. Toss four fair coins and find the probability of three or more heads.

25. Roll a fair die twice and find the probability of at least one 4.

26. Roll two fair dice and find the probability that the sum of the two numbers is even.

27. Roll two fair dice, one after the other, and find the probability that the first number is larger than the second number.

28. Roll two fair dice and find the probability that the minimum of the two numbers will be greater than 4.

29. In Example 11, we considered a cross between two pea plants, each of genotype Cc. Find the probability that a randomly chosen seed from this cross has white flowers.

30. In Example 11, we considered a cross between two pea plants, each of genotype Cc. Now we cross a pea plant of genotype cc with a pea plant of genotype Cc.

(a) What are the possible outcomes of this crossing?

(b) Find the probability that a randomly chosen seed from this crossing results in red flowers.

31. Suppose that two parents are of genotype Aa. What is the probability that their offspring is of genotype Aa? (Assume Mendel's first law.)

32. Suppose that one parent is of genotype AA and the other is of genotype Aa. What is the probability that their offspring is of genotype AA? (Assume Mendel's first law.)

33. A family has three children. Assuming a 1:1 sex ratio, what is the probability that all of the children are girls?

34. A family has three children. Assuming a 1:1 sex ratio, what is the probability that at least one child is a boy?

35. A family has four children. Assuming a 1:1 sex ratio, what is the probability that no more than two children are girls?

In Problems 36–37, we discuss the inheritance of red–green color blindness. Color blindness is an X-linked inherited disease. A woman who carries the color blindness gene on one of her X chromosomes, but not on the other, has normal vision. A man who carries the gene on his only X chromosome is color blind.

36. If a woman with normal vision who carries the color blindness gene on one of her X chromosomes has a child with a man who has normal vision, what is the probability that their child will be color blind?

37. If a woman with normal vision who carries the color blindness gene on one of her X chromosomes has a child with a man who is red–green color blind, what is the probability that their child has normal vision?

38. Cystic fibrosis is an autosomal recessive disease, which means that two copies of the gene must be mutated for a person to be affected. Assume that two unaffected parents who each carry a single copy of the mutated gene have a child. What is the probability that the child is affected?

39. An urn contains three red and two blue balls. You remove two balls without replacement. What is the probability that the two balls are of a different color?

40. An urn contains five blue and three green balls. You remove three balls from the urn without replacement. What is the probability that at least two out of the three balls are green?

41. You select 2 cards without replacement from a standard deck of 52 cards. What is the probability that both cards are spades?

42. You select 5 cards without replacement from a standard deck of 52 cards. What is the probability that you get four aces?

43. An urn contains four green, six blue, and two red balls. You take three balls out of the urn without replacement. What is the probability that all three balls are of different colors?

44. An urn contains three green, five blue, and four red balls. You take three balls out of the urn without replacement. What is the probability that all three balls are of the same color?

45. Four cards are drawn at random without replacement from a standard deck of 52 cards. What is the probability of at least one ace?

46. Four cards are drawn at random without replacement from a standard deck of 52 cards. What is the probability of exactly one pair?

47. Thirteen cards are drawn at random without replacement from a standard deck of 52 cards. What is the probability that all are red?

48. Four cards are drawn at random without replacement from a standard deck of 52 cards. What is the probability that all are of different suits?

49. Five cards are drawn at random without replacement from a standard deck of 52 cards. What is the probability of exactly two pairs?

50. Five cards are drawn at random without replacement from a standard deck of 52 cards. What is the probability of three of a kind and a pair (for instance, Q Q Q 3 3)? (This is called a *full house* in poker.)

51. A lake contains an unknown number of fish, denoted by N. You capture 100 fish, mark them, and subsequently release them. Later, you return and catch 10 fish, 3 of which are marked.

(a) Find the probability that exactly 3 out of 10 fish you just caught will be marked. This probability will be a function of N, the unknown number of fish in the lake.

(b) Find the value of N that maximizes the probability you computed in (a), and show that this value agrees with the value we computed in Example 13.

12.3 Conditional Probability and Independence

Before we define *conditional probability* and *independence*, we will illustrate these concepts by using the Mendelian crossing of peas that we considered in the previous section to study flower color inheritance.

Assume that two parent pea plants are of genotype Cc. Suppose you know that the offspring of the crossing $Cc \times Cc$ has red flowers. What is the probability that it is of genotype CC? We can find this probability by noting that one of the three equally likely possibilities that produce red flowers [namely, (C, C), (C, c), and (c, C) if we list the types according to maternal and paternal contributions as in Example 11 of the previous section] is of type CC. Hence, the probability that the offspring is of genotype CC is 1/3. Such a probability, conditioned on some prior knowledge (such as flower color of offspring), is called a **conditional probability**.

Suppose now that the paternally transmitted gene in the offspring of the crossing $Cc \times Cc$ is of type C. What is the probability that the maternally transmitted gene in the offspring is of type c? To answer this question, we note that the paternal gene has no impact on the choice of the maternal gene in this case. The probability that the maternal gene is of type c is therefore 1/2. We say that the maternal gene is **independent** of the paternal gene: Knowing which of the paternal genes was chosen does not change the probability of the maternal gene.

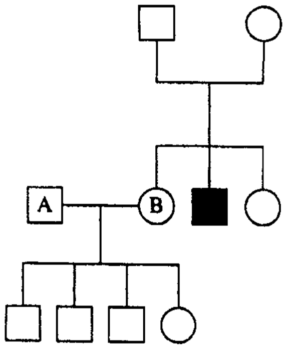

Figure 12.21 The pedigree of a family in which one member suffers from hemophilia. Squares indicate males, circles females. The black square shows an afflicted individual.

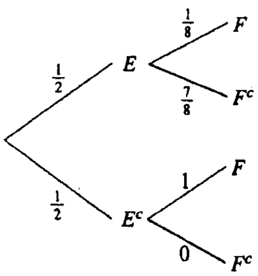

Figure 12.22 The sample space is partitioned into two sets—E and E^c—where E is the event that the individual B is a carrier of the hemophilia gene. Based on whether or not B is a carrier, the probability of the event that all three sons are healthy (F) can be computed as shown.

Pedigrees of families show family relationships among individuals and are indispensable tools for tracing diseases of genetic origin. In a pedigree, males are denoted by squares, females by circles; blackened symbols denote individuals who suffer from the disease that is tracked by the pedigree. Figure 12.21 shows the pedigree of a family in which one male (the black square) suffers from hemophilia. We will use this pedigree to determine the probability that individual B is a carrier of the disease given that all three sons of A and B are disease free.

We see from the pedigree that B has a hemophilic brother. Therefore, B's mother must be a carrier. There is a 50% chance that a sister of the affected individual is a carrier. We denote the event that B is a carrier by E. Then $P(E) = 1/2$. Now, assume that we are told that B has three sons with an unaffected male (A). If F denotes the event that all three sons are healthy, then

$$P(F \mid E) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

since if B is a carrier, each son has probability 1/2 of not inheriting the disease gene and thus being healthy.

We can use the Bayes formula to compute the probability that B is a carrier given that none of her three sons has the disease:

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{P(F \mid E)P(E)}{P(F)}$$

To compute the denominator, we must use the law of total probability, as illustrated in the tree diagram in Figure 12.22.

We find that

$$P(F) = \frac{1}{2} \cdot \frac{1}{8} + \frac{1}{2} \cdot 1 = \frac{9}{16}$$

Therefore, using the Bayes formula, we obtain

$$P(E \mid F) = \frac{\frac{1}{8} \cdot \frac{1}{2}}{\frac{9}{16}} = \frac{1}{9}$$

or, in words, on the basis of the pedigree, the probability that B is a carrier of the gene causing hemophilia given that none of her three sons is symptomatic for the disease is 1/9.

Section 12.3 Problems

■ **12.3.1**

1. Suppose you draw 2 cards from a standard deck of 52 cards. Find the probability that the second card is a spade given that the first card is a club.

2. Suppose you draw 2 cards from a standard deck of 52 cards. Find the probability that the second card is a spade given that the first card is a spade.

3. Suppose you draw 3 cards from a standard deck of 52 cards. Find the probability that the third card is a club given that the first two cards are spades.

4. Suppose you draw 3 cards from a standard deck of 52 cards. Find the probability that the third card is a club given that the first two cards are clubs.

5. An urn contains five blue and six green balls. You take two balls out of the urn, one after the other, without replacement. Find the probability that the second ball is green given that the first ball is blue.

6. An urn contains five green, six blue, and four red balls. You take three balls out of the urn, one after the other, without replacement. Find the probability that the third ball is green given that the first two balls were red.

7. A family has two children. The younger one is a girl. Find the probability that the other child is a girl as well.

8. A family has two children. One of their children is a girl. Find the probability that both children are girls.

9. You roll two fair dice. Find the probability that the first die is a 4 given that the sum is 7.

10. You roll two fair dice. Find the probability that the first die is a 5 given that the minimum of the two numbers is a 3.

11. You toss a fair coin three times. Find the probability that the first coin is heads given that at least one head occurred.

12. You toss a fair coin three times. Find the probability that at least two heads occurred given that the second toss resulted in heads.

13. You toss a fair coin four times. Find the probability that four heads occurred given that the first toss and the third toss resulted in heads.

14. You toss a fair coin four times. Find the proability of no more than three heads given that at least one toss resulted in heads.

■ 12.3.2

15. A screening test for a disease shows a positive test result in 90% of all cases when the disease is actually present and in 15% of all cases when it is not. Assume that the prevalence of the disease is 1 in 100. If the test is administered to a randomly chosen individual, what is the probability that the result is negative?

16. A screening test for a disease shows a positive result in 92% of all cases when the disease is actually present and in 7% of all cases when it is not. Assume that the prevalence of the disease is 1 in 600. If the test is administered to a randomly chosen individual, what is the probability that the result is positive?

17. A patient underwent a diagnostic test for hypothyroidism. The diagnostic test correctly identifies patients who in fact have the disease in 93% of the cases and correctly identifies healthy patients in 81% of the cases. If 4 in 100 individuals have the disease, what is the probability that a test comes back negative?

18. A screening test for a disease shows a positive test result in 95% of all cases when the disease is actually present and in 20% of all cases when it is not. When the test was administered to a large number of people, 21.5% of the results were positive. What is the prevalence of the disease?

19. A drawer contains three bags numbered 1–3, respectively. Bag 1 contains three blue balls, bag 2 contains four green balls, and bag 3 contains two blue balls and one green ball. You choose one bag at random and take out one ball. Find the probability that the ball is blue.

20. A drawer contains six bags numbered 1–6, respectively. Bag i contains i blue balls and 2 green balls. You roll a fair die and then pick a ball out of the bag with the number shown on the die. What is the probability that the ball is blue?

21. You pick 2 cards from a standard deck of 52 cards. Find the probability that the second card is an ace. Compare this with the probability that the first card is an ace.

22. You pick 3 cards from a standard deck of 52 cards. Find the probability that the third card is an ace. Compare this with the probability that the first card is an ace.

23. Suppose that you have a batch of red-flowering pea plants of which 40% are of genotype CC and 60% of genotype Cc. You pick one plant at random and cross it with a white-flowering pea plant. Find the probability that the offspring of this crossing will have white flowers.

24. Suppose that you have a batch of red- and white-flowering pea plants, and suppose also that all three genotypes CC, Cc, and cc are equally represented in the batch. You pick one plant at random and cross it with a white-flowering pea plant. What is the probability that the offspring will have red flowers?

25. A bag contains two coins, one fair and the other with two heads. You pick one coin at random and flip it. Find the probability that the outcome is heads.

26. A drug company claims that a new headache drug will bring instant relief in 90% of all cases. If a person is treated with a placebo, there is a 20% chance that the person will feel instant relief. In a clinical trial, half the subjects are treated with the new drug and the other half receive the placebo. If an individual from this trial is chosen at random, what is the probability that the person will have experienced instant relief?

■ 12.3.3

27. You are dealt 1 card from a standard deck of 52 cards. If A denotes the event that the card is a spade and if B denotes the event that the card is an ace, determine whether A and B are independent.

28. You are dealt 2 cards from a standard deck of 52 cards. If A denotes the event that the first card is an ace and B denotes the event that the second card is an ace, determine whether A and B are independent.

29. An urn contains five green and six blue balls. You take two balls out of the urn, one after the other, without replacement. If A denotes the event that the first ball is green and B denotes the event that the second ball is green, determine whether A and B are independent.

30. An urn contains four green and three blue balls. You take one ball out of the urn, note its color, and replace it. You then take a second ball out of the urn, note its color, and replace it. If A denotes the event that the first ball is green and B denotes the event that the second ball is green, determine whether A and B are independent.

31. Assume a 1:1 sex ratio. A family has three children. Find the probability of the event
(a) $A = \{\text{all children are girls}\}$ **(b)** $B = \{\text{at least one boy}\}$
(c) $C = \{\text{at least two girls}\}$ **(d)** $D = \{\text{at most two boys}\}$

32. Assume that 20% of a very common insect species in your study area is parasitized. Assume that insects are parasitized independently of each other. If you collect 10 specimens of this species, what is the probability that no more than 2 specimens in your sample are parasitized?

33. A multiple-choice question has four choices, and a test has a total of 10 multiple-choice questions. A student passes the test only if he or she answers all questions correctly. If the student guesses the answers to all questions randomly, find the probability that he or she will pass.

34. Assume that A and B are disjoint and that both events have positive probability. Are they independent?

35. Assume that the probability that an insect species lives more than five days is 0.1. Find the probability that, in a sample of size 10 of this species, at least one insect will still be alive after five days.

36. **(a)** Use a Venn diagram to show that

$$(A \cup B)^c = A^c \cap B^c$$

(b) Use your result in (a) to show that if A and B are independent, then A^c and B^c are independent.

(c) Use your result in (b) to show that if A and B are independent, then

$$P(A \cup B) = 1 - P(A^c)P(B^c)$$

■ 12.3.4

37. A screening test for a disease shows a positive result in 95% of all cases when the disease is actually present and in 10% of all cases when it is not. If the prevalence of the disease is 1 in 50 and an individual tests positive, what is the probability that the individual actually has the disease?

38. A screening test for a disease shows a positive result in 95% of all cases when the disease is actually present and in 10% of all cases when it is not. If a result is positive, the test is repeated. Assume that the second test is independent of the first test. If the prevalence of the disease is 1 in 50 and an individual tests positive twice, what is the probability that the individual actually has the disease?

39. A bag contains two coins, one fair and the other with two heads. You pick one coin at random and flip it. What is the probability that you picked the fair coin given that the outcome of the toss was heads?

40. You pick 2 cards from a standard deck of 52 cards. Find the probability that the first card was a spade given that the second card was a spade.

41. Suppose a woman has a hemophilic brother and one healthy son. Suppose furthermore that neither her mother nor her father were hemophilic but that her mother was a carrier for hemophilia. Find the probability that she is a carrier of the hemophilia gene.

The pedigree in Figure 12.23 shows a family in which one member (III-4) is hemophilic. In Problems 42 and 43, refer to this pedigree.

42. **(a)** Given the pedigree, find the probability that the individual I-2 is a carrier of the hemophilia gene.

(b) Given the pedigree, find the probability that II-3 is a carrier of the hemophilia gene.

43. **(a)** Given the pedigree, find the probability that II-3 is a carrier of the hemophilia gene.

(b) Given the pedigree, find the probability that III-2 is a carrier of the hemophilia gene.

(c) Given the pedigree, find the probability that II-2 is a carrier of the hemophilia gene.

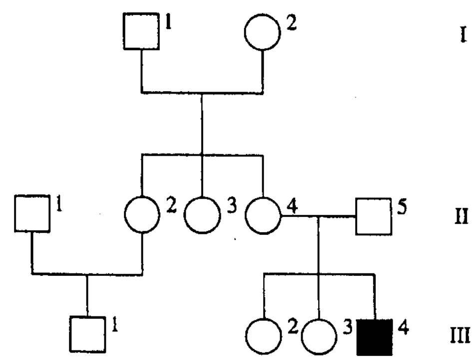

Figure 12.23 The pedigree for Problems 42 and 43. The solid black square (individual III-4) represents an afflicted male.

■ 12.4 Discrete Random Variables and Discrete Distributions

Outcomes of random experiments frequently are real numbers, such as the number of heads in a coin-tossing experiment, the number of seeds produced in a crossing between two plants, or the life span of an insect. Such numerical outcomes can be described by **random variables.** A random variable is a function from the sample space Ω into the set of real numbers. Random variables are typically denoted by X, Y, or Z, or other capital letters chosen from the end of the alphabet. For instance,

$$X : \Omega \to \mathbf{R}$$

describes the random variable X as a map from the sample space Ω into the set of real numbers.

Random variables are classified according to their range. If X takes on a discrete set of values (finite or infinite), X is called a **discrete random variable.** If X takes on a continuous range of values—for instance, values that range over an interval—X is called a **continuous random variable.** Discrete random variables are the topic of this section; continuous random variables are the topic of the next section.

■ 12.4.1 Discrete Distributions

In the first two examples in this section, we look at random variables that take on a discrete set of values. In the first example, this set is finite; in the second example, the set is infinite.

EXAMPLE 1 Toss a fair coin three times. Let X be a random variable that counts the number of heads in each outcome. The sample space is

$$\Omega = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

and the random variable

$$X : \Omega \to \mathbf{R}$$

takes on values 0, 1, 2, or 3. For instance,

$$X(HHH) = 3 \quad \text{or} \quad X(TTH) = 1 \quad \text{or} \quad X(TTT) = 0$$ ■

EXAMPLE 2 Toss a fair coin repeatedly until the first time heads appears. Let Y be a random variable that counts the number of trials until the first time heads shows up. The sample space is

$$\Omega = \{H, TH, TTH, TTTH, \dots\}$$