Math 17C
Kouba
Discussion Sheet 2

1.) Evaluate the following limits or determine that the limit does not exist.

a.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2-4}{x+y+2}$ b.) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{xy-y-2x+2}{x-1}$

c.) $\displaystyle\lim_{(x,y)\to(2,2)} \frac{x+y-4}{\sqrt{x+y}-2}$ d.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$

e.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3}{x^3+y^3}$ f.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ g.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

2.) Compute z_x and z_y for each of the following functions.

a.) $z = xy^2 + \ln x + e^y + 5$ b.) $z = xe^{2y}\arctan x$ c.) $z = \sqrt{x-y^2}$

d.) $z = \dfrac{x^3}{y^2} + \sin(xy)$ e.) $z = \dfrac{x+4}{x^2+y^2}$ f.) $z = \{e^{x^2y} + \tan(3y+4x)\}^5$

g.) $z = y^{1+x^3}$

3.) Determine functions z whose partial derivatives are given, or state that this is impossible.

a.) $z_x = 2x$ and $z_y = 3y^2+1$ b.) $z_x = xy^2 - y$ and $z_y = x^2y - x$

c.) $z_x = e^x y - 1$ and $z_y = e^x - x$ d.) $z_x = y^2\cos(xy)$ and $z_y = xy\cos(xy) + \sin(xy)$

4.) Plane A, parallel to the xz-plane, and plane B, parallel to the yz-plane, pass through the surface determined by the equation $z = xy^2 - x^3 + 7$. Both planes include the point $(1,0,6)$, which lies on the surface.

a.) Determine the slope of the line tangent to the surface at the point $(1,0,6)$ if the line lies in

i.) plane A.

ii.) plane B.

b.) Determine an equation of the plane tangent to the surface at the point $(1,0,6)$.

5.) Determine an equation of the plane tangent to the surface at the given point.

a.) $z = x^2 + y^2$, point $(1,-1)$

b.) $z = xy$, point $(3,4)$

c.) $z = (xy^4+1)^3$, point $(2,-1)$

6.) Determine the linearization, $L(x)$, for $f(x) = x^2(x-1)$ at $x = 2$. Use $L(x)$ to estimate the value of f at $x = 1.9$.

7.) Determine the linearization, $L(x,y)$, for $f(x,y) = x^2 + 2y^2$ at $(x,y) = (1,-1)$. Use $L(x,y)$ to estimate the value of f at the point $(x,y) = (1.1,-0.9)$.

8.) Determine the linearization, $L(x,y)$, for $f(x,y) = \dfrac{x}{y^2}$ at $(x,y) = (3,2)$. Use $L(x,y)$ to estimate the value of f at the point $(x,y) = (2.9, 1.8)$.

9.) Write a precise ϵ/δ proof for each limit.

a.) $\displaystyle\lim_{(x,y)\to(0,0)} (x+y) = 0$

b.) $\displaystyle\lim_{(x,y)\to(2,-1)} (x+y) = 1$

c.) $\displaystyle\lim_{(x,y)\to(1,-1)} (x-y+3) = 5$

d.) $\displaystyle\lim_{(x,y)\to(0,0)} (x^2+3y) = 0$

e.) $\displaystyle\lim_{(x,y)\to(-1,1)} (x^2+3y) = 4$

f.) $\displaystyle\lim_{(x,y)\to(1,1)} (x^2-xy-x) = -1$

g.) $\displaystyle\lim_{(x,y)\to(0,0)} (x^2+2y^2) = 0$

h.) $\displaystyle\lim_{(x,y)\to(3,4)} (x^2+y^2) = 25$

i.) $\displaystyle\lim_{(x,y)\to(0,0)} \sqrt{4-x^2-y^2} = 2$

j.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x+3y}{x^2+y^2+1} = 0$

10.) Verify that the function $f(x,y) = \begin{cases} (x^2+y^2)^{3/2} & , \quad (x,y) \neq (0,0) \\ 125 & , \quad (x,y) = (0,0) \end{cases}$

a.) is NOT continuous at $(0,0)$.

b.) is continuous at $(3,4)$.

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"Whether you think you can, or that you can't, you are usually right." – Henry Ford (1863-1947)