Math 17C
Kouba
Discussion Sheet 3

1.) Find the Jacobi Matrix for each function.

a.) $f(x,y) = \begin{pmatrix} 3x+4y \\ 2x-5y \end{pmatrix}$

b.) $f(x,y) = \begin{pmatrix} x+e^y \\ \ln(x-y) \end{pmatrix}$

c.) $f(x,y) = \begin{pmatrix} x/y \\ x^3 \sin y \end{pmatrix}$

d.) $f(x,y) = \begin{pmatrix} \tan(xy) \\ \cos(5x-y) \end{pmatrix}$

e.) $f(x,y) = \begin{pmatrix} \sqrt{x^2+y^2} \\ x\sqrt{y} \end{pmatrix}$

2.) Find the linearization $L(x,y)$ of $f(x,y) = \begin{pmatrix} x^2+3y \\ xy^2 \end{pmatrix}$ at $(1,-1)$.

a.) Compute $f(0.9,-1.2)$ and $L(0.9,-1.2)$. What do you conclude ?
b.) Compute $f(3,2)$ and $L(3,2)$. What do you conclude ?

3.) Assume that $z = x^2y + xy^2$, $x = t^3$, and $y = \sqrt{t}$. Compute $\frac{dz}{dt}$ when $t = 1$.

4.) Assume that $z = \ln(xy) + e^{2x+3y}$, $x = f(t)$, and $y = g(t)$. If $f(0) = -1$, $f'(0) = 2$, $g(0) = 3$, and $g'(0) = 1$, then what is the value of $\frac{dz}{dt}$ when $t = 0$?

5.) Assume that y is a function of x and $x^2y^3 + ye^{2x} = \ln y$. Use the partial derivative "shortcut" to determine $\frac{dy}{dx}$.

6.) A Madagascar hissing cockroach is walking on the plane $x + 2y + 3z = 6$ above the circle $x^2 + y^2 = 1$. Assume that the roach's position (x, y, z) (distance measured in inches) at time t seconds is determined by $x = \cos t$ and $y = \sin t$. Determine the roach's rate of change of elevation $\left(\frac{dz}{dt}\right)$ when $t = 0$ seconds; $t = \pi$ seconds.

7.) Compute the derivative of $f(x,y) = \ln(2x+3y)$ at the point $P = (2,0)$ in the direction of vector $A = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

8.) Consider the function $f(x,y) = xy^3$ and the point $P = (2,1)$. Determine all unit vectors u so that $D_{\vec{u}} f(2,1)$ is

a.) as large as possible (f increases most rapidly).
b.) as small as possible (f decreases most rapidly).
c.) equal to zero. d.) equal to 1.

9.) Assume that the surface of a volcanic mountain is given by the function $z = 5e^{-(x^2+y^2)}$, where z is measured in miles. Sketch this mountain in 3D-Space. Assume you are standing on this mountain at the point $(x,y) = (1/2, \sqrt{3}/2)$. What is your elevation ?

a.) Find the SLOPE of the mountain at this point in the

i.) positive x-direction.
ii.) positive y-direction.
iii.) negative x-direction.
iv.) direction of vector $\vec{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

b.) In what direction will the SLOPE at this point be

i.) largest ? What is the value of this slope ?
ii.) smallest ? What is the value of this slope ?

10.) Consider the surface given by $x^2 + y^2 + z^2 = 169$ and the point $P = (3,4,12)$ on the surface. Find equations for

a.) the plane tangent to the surface at point P.
b.) the line normal (perpendicular) to the surface at point P.

11.) Consider the surface (hyperbolic paraboloid or saddle) given by $f(x,y) = 3x^2 - 2y^2 + 5$ and the point $P = (2,3,-1)$ on the surface. Find equations for

a.) the plane tangent to the surface at point P.
b.) the line normal (perpendicular) to the surface at point P.

12.) Let $f(x,y) = xy^2 + 2x - 3y$. Use a linearization $L(x)$ at the point $(0,0)$ to estimate the value of $f(0.2, -0.1)$.

13.) Find and classify critical points as determining relative maximums, relative minimums, or saddle points.

a.) $z = 3x^2 - 6xy + y^2 + 12x - 16y + 1$ b.) $z = x^2y - x^2 - 2y^2$
c.) $z = x^2 - 8\ln(xy) + y^2$ d.) $z = 3x^2y - 6x^2 + y^3 - 6y^2$

14.) Find the point on the plane $x + 2y + 3z = 6$ nearest the origin.

15.) Determine the dimensions and minimum surface area of a closed rectangular box with volume 8 ft.^{3}

16.) Determine the point on the sphere $x^2 + y^2 + z^2 = 4$ which is a.) nearest the point $(1, -1, 1)$. b.) farthest from the point $(1, -1, 1)$.

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"If you judge people, you have no time to love them." - Mother Teresa