Math 17C
Kouba
Discussion Sheet 5

1.) The position (x_1, x_2) of a particle at time t is given parametrically by each of the following. Eliminate t and write each as an equation in only x_1 and x_2 . Then sketch the graph of the path in the x_1x_2-plane, indicating the direction of motion of the particle.

a.) $\begin{cases} x_1 = 3t + 2 \\ x_2 = 2t - 5, \end{cases}$ for $-\infty < t < \infty$.

b.) $\begin{cases} x_1 = \ln t \\ x_2 = (\ln t)^3 - 2(\ln t)^2, \end{cases}$ for $t > 0$.

c.) $\begin{cases} x_1 = t^2 \\ x_2 = t^6 - 2t^4, \end{cases}$ for $-\infty < t < \infty$..

d.) $\begin{cases} x_1 = 2 + \sqrt{t} \\ x_2 = \sqrt{4 - t}, \end{cases}$ for $0 \le t \le 4$.

e.) $\begin{cases} x_1 = \cos t \\ x_2 = \sin t - 3, \end{cases}$ for $0 \le t \le 2\pi$.

f.) (Challenging) $\begin{cases} x_1 = t^2 - 2t \\ x_2 = t^2 + t, \end{cases}$ for $-\infty < t < \infty$.

2.) Use the parametric graphing function on a graphing calculator to plot the following path. Then find a unit vector tangent to the path, the direction of motion, and the speed of motion when a.) $t = \pi$ b.) $t = 7\pi/2$.

$\begin{cases} x_1 = t \cos t \\ x_2 = t \sin t, \end{cases}$ for $0 \le t \le 4\pi$.

3.) Write the following system of differential equations in matrix (vector) form.

$$\frac{dx_1}{dt} = 7x_2$$
$$\frac{dx_2}{dt} = 3x_1 - x_2$$

4.) Write the following system of differential equations in parametric form.

$$X' = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix} X$$

5.) Show that $\begin{cases} x_1 = 5\cos 3t \\ x_2 = 4\cos 3t + 3\sin 3t \end{cases}$ solves the following system of differential equations :

$$\frac{dx_1}{dt} = 4x_1 - 5x_2$$
$$\frac{dx_2}{dt} = 5x_1 - 4x_2$$

6.) Show that $X = \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^t + \begin{pmatrix} 4 \\ -4 \end{pmatrix} te^t$ solves the following system of differential equations : $X' = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} X$

7.) (Creating a direction field) Consider the following system of differential equations. For each of the following pairs of points (x_1, x_2) set up a table to indicate the slope, direction vector, and speed at that point. On an x_1x_2-coordinate system plot the direction vector at each point and indicate the relative length (speed) of each vector. Use the following points : (1,1), (1,2), (1,0), (1,-1), (1,-2), (0,0), (0,1,), (0,2),(0, -1), (0,-2), (-1,1), (-1,2), (-1,0), (-1,-1), (-1,-2), (3,-3), (4,2)

$$\frac{dx_1}{dt} = -2x_1 + x_2$$
$$\frac{dx_2}{dt} = x_1 - 2x_2$$

8.) Find the general solution to each of the following systems of differential equations. Write your answer in matrix (vector) form.

a.) $X' = \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix} X$ b.) $X' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} X$

c.) $X' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} X$ d.) $X' = \begin{pmatrix} -3 & 3/4 \\ -5 & 1 \end{pmatrix} X$

9.) Solve the following system of differential equations with initial conditions. Write your answer in matrix (vector) form and parametric form.

$$\frac{dx_1}{dt} = x_1 + 2x_2 \ , \ x_1(0) = 5$$
$$\frac{dx_2}{dt} = 4x_1 + 3x_2 \ , \ x_2(0) = -2$$

10.) The point $(0,0)$ is an equilibrium for each of the systems in problem 8.) For each system determine if $(0,0)$ is an unstable or stable equilibrium. Then categorize $(0,0)$ as a saddle, sink, or source.

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"If you judge people, you have no time to love them." – Mother Teresa