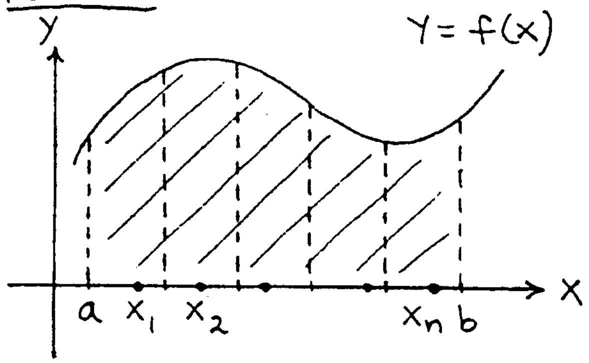
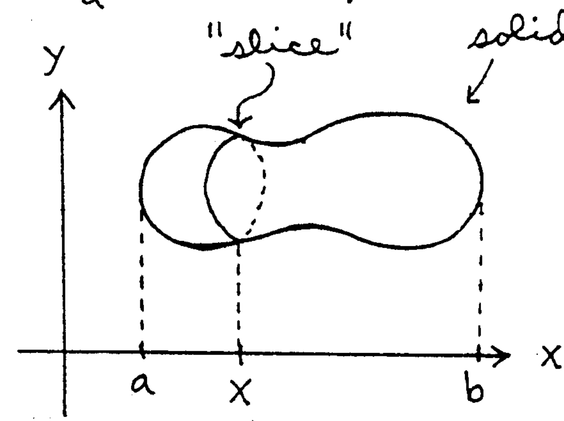
Math 17C
Kouba
Double Integrals

Recall :



Divide interval [a, b] into n equal parts each of length $\frac{b-a}{n}$. Then the $\boxed{\text{area}}$ of the shaded region is given by

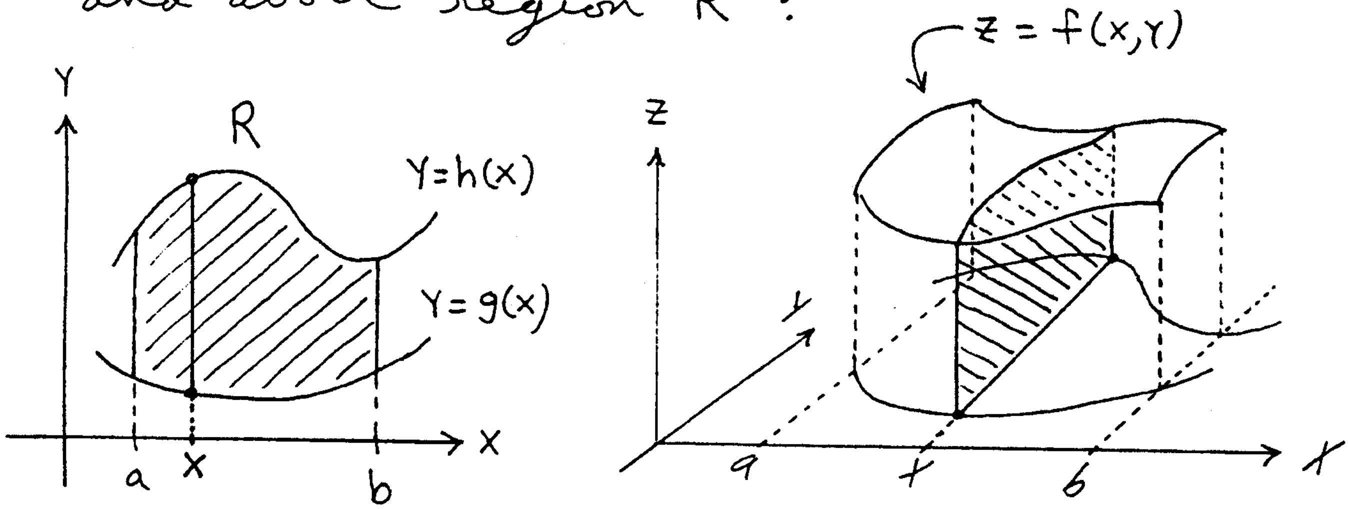
$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\frac{b-a}{n} = \int_a^b f(x)\,dx .$$



Fact : If A(x) is the area of a "slice" of a solid taken perpendicular to the x-axis at x, then the $\boxed{\text{volume}}$ of the solid is given by

$$\text{Volume} = \int_a^b A(x)\,dx .$$

Double Integral : Consider region R bounded by $a \le x \le b$ and $g(x) \le y \le h(x)$ and let $z = f(x, y)$ be a surface defined on R. We seek to compute the $\boxed{\text{volume}}$ of the solid region lying below the surface and above region R :



Pick an x-value and make a slice perpendicular to the x-axis. Let $A(x)$ be the area of this slice. Then

$$A(x) = \int_{g(x)}^{h(x)} f(x,y)\, dy$$

and the volume of the solid is

$$\text{Volume} = \int_a^b A(x)\, dx \,, \text{ or}$$

$$\boxed{\text{Volume} = \int_a^b \int_{g(x)}^{h(x)} f(x,y)\, dy\, dx} \,.$$