Math 17C
Kouba
Worksheet 1

1.) Write a precise ϵ/δ proof for each limit.

a.) $\lim\limits_{(x,y)\to(0,0)} \sqrt{x^2 + y^2} = 0$

b.) $\lim\limits_{(x,y)\to(0,0)} (2x^2 + 3y^2) = 0$

c.) $\lim\limits_{(x,y)\to(1,-1)} (x + y) = 0$

d.) $\lim\limits_{(x,y)\to(0,2)} (2x - y + 1) = -1$

e.) $\lim\limits_{(x,y)\to(3,2)} xy = 6$

2.) Write a precise ϵ/δ proof for each statement.

a.) $f(x, y) = x^2 - y^2$ is continuous at $(0, 0)$.

b.) $f(x, y) = \sqrt{x^2 + y^2}$ is continuous at $(2, 0)$.

3.) Show that the function $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & , \quad (x, y) \neq (0, 0) \\ 0 & , \quad (x, y) = (0, 0) \end{cases}$

is NOT continuous at $(0, 0)$.

4.) Show that the function $f(x, y) = \begin{cases} 1 + x + y & , \quad (x, y) \neq (0, 0) \\ 2 & , \quad (x, y) = (0, 0) \end{cases}$

a.) is NOT continuous at $(0, 0)$.

b.) is continuous at $(4, -3)$.