

Math 17C

Kouba

Exam 1

(Practice)

Please PRINT your name here : \_\_\_\_\_

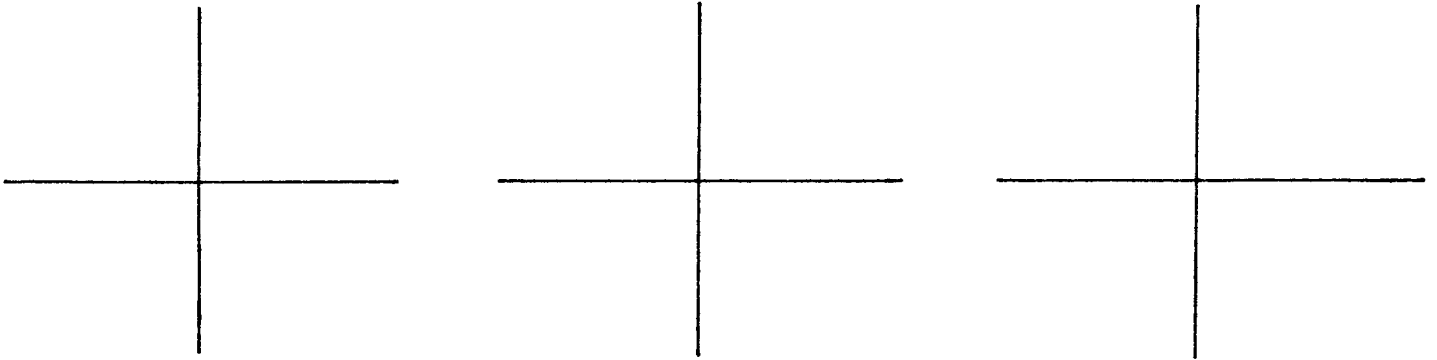
Your Exam ID Number \_\_\_\_\_

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.
2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. COPYING ANSWERS FROM ANOTHER STUDENT'S EXAM IS A VIOLATION OF THE UNIVERSITY HONOR CODE. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.
3. YOU MAY USE A CALCULATOR ON THIS EXAM.
4. No notes, books, or classmates may be used as resources for this exam.
5. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will receive LITTLE or NO credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.
6. You have until 8:50 a.m. sharp to finish the exam. PLEASE STOP WRITING IMMEDIATELY when time is called and close your exam.
7. Make sure that you have 7 pages including the cover page.

1.) Consider the function given by  $z = x^2 + y^2 - 4$  and its graph in 3D-space.

a.) (4 pts.) Determine all possible intercepts for this equation.

b.) (4 pts.) Determine and sketch the  $xy$ -trace,  $xz$ -trace, and  $yz$ -trace.



c.) (4 pts.) Sketch level curves on the same axes for  $z = -3, 0,$  and  $5$ .

d.) (4 pts.) Sketch the surface in 3D-space.

2.) (9 pts.) Let  $z = 3x + \ln(x^2 + y)$ . Compute the partial derivatives  $z_x$ ,  $z_y$ , and  $z_{xx}$ . DO NOT SIMPLIFY your answers.

3.) (11 pts.) Find and classify (relative maximum value, relative minimum value, or saddle point) each critical point for the following function :

$$f(x, y) = x^3 - y^3 + 3xy$$

4.) (9 pts.) Evaluate the following limit :  $\lim_{(x,y) \rightarrow (1,-1)} \frac{1 - \sqrt{x+y+1}}{x+y}$

5.) (9 pts.) Verify that the following limit does not exist :  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6}$

6.) Let  $f(x, y) = 4 - \sqrt{y - x^2}$ .

a.) (5 pts.) Determine and sketch the domain of  $f$  in 2D-space.

b.) (5 pts.) State the range of  $f$ . BRIEFLY explain how you get your answer.

7.) (9 pts.) Give a precise  $\epsilon, \delta$ -proof that  $\lim_{(x,y) \rightarrow (1,0)} (xy + x) = 1$

8.) (9 pts.) Consider the surface given by  $z = xy^2 - 3y$  and point  $P = (1, -1, 4)$  on this surface. Determine an equation for the plane tangent to this surface at point  $P$ .

9.) (9 pts.) Consider the function  $f(x, y) = x^2 - y^2$ , the point  $P = (2, -1)$ , and vector  $\vec{v} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ . Compute the directional derivative of  $f$  in the direction of  $\vec{v}$  at point  $P$ .

10.) (9 pts.) Find the linearization  $L(x)$  of  $f(x, y) = \begin{pmatrix} 3x + y^2 \\ x^2y \end{pmatrix}$  at  $(1, -1)$ .

The following EXTRA CREDIT PROBLEM is worth 10 points. This problem is OPTIONAL.

1.) Consider the graph of  $z = \sqrt{x}$  in the  $xz$ -plane. Find an equation for the surface created by revolving this graph about the  $z$ -axis.