Math 17C
Kouba
Exam 3 (Practice)

Please PRINT your name here : KEY

Your Exam ID Number ----------

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO HAVE ANOTHER STUDENT TAKE YOUR EXAM FOR YOU. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 6 pages, including the cover page.

6. You have until 8:50 a.m. sharp to finish the exam. Failure to stop working on your exam when time is called may lead to points deducted from your total score. Thank you for your cooperation.

1.) (7 pts.) How many distinct 11-letter permutations are there of the letters in the word MISSISSIPPI ?

$$\frac{11!}{4!\,4!\,2!} = 34{,}650$$

($4!$: S's, $4!$: I's, $2!$: P's)

2.) (7 pts.) How many distinct factors does 9000 have ?

$9000 = 9 \cdot 1000 = 3^2 \cdot (2 \cdot 5)^3 = 2^3 \cdot 3^2 \cdot 5^3$

$$\underset{\#\ 2\text{'s}}{4} \cdot \underset{\#\ 3\text{'s}}{3} \cdot \underset{\#\ 5\text{'s}}{4} = 48 \text{ factors}$$

3.) (7 pts) A group of 20 students includes 11 girls and 9 boys. In how many ways can a committee of 6 students be chosen if the committee must have exactly 4 girls and 2 boys and each committee member will be assigned exactly one of the following 6 positions – president, vice-president, secretary, treasurer, editor, fundraiser ?

$$\underset{\text{pick girls}}{C(11,4)} \cdot \underset{\text{pick boys}}{C(9,2)} \cdot \underset{P}{6} \cdot \underset{V.P.}{5} \cdot \underset{S}{4} \cdot \underset{T}{3} \cdot \underset{E}{2} \cdot \underset{F}{1}$$

$= 330 \cdot 36 \cdot 720 = 8{,}553{,}600$

4.) (7 pts) Your chemistry professor is to assign grades of A, B, C, D, or F in a class of 250 students. How many different grade distributions (total number of A's, B's, C's, D's, and F's: for example, 40 A's, 75 B's, 90 C's, 40 D's and 5 F's) are possible ?

5 choices to 4 partitions →

$C(250+4, 4) = 169,362,501$

5.) (7 pts) We are going to select 12 donuts from among sprinkles, caramel, chocolate, and maple flavors (all equally likely). What is the probability that we select exactly 3 maple and 1 chocolate donut ?

M M M C | _ _ _ □ _ _ _ _ _

TOTAL: $C(12+3, 3) = C(15, 3)$

pick 8 from 2 flavors: $C(8+1, 1) = C(9, 1)$

$$P(3M, 1C) = \frac{C(9,1)}{C(15,3)} = \frac{9}{455}$$

6.) Let sample space $\Omega = \{1, 2, 3, 4, 5, 6\}$ with all outcomes equally likely. Let events $A = \{\ \}$ (empty set), $B = \{2, 4, 6\}$, and $C = \{2, 3, 4, 6\}$.

a.) (6 pts.) LIST the elements in sets $A \cup B$, $B \cap C$, and A^c .

$A \cup B = \{2,4,6\}$, $B \cap C = \{2,4,6\}$, $A^c = \{1,2,3,4,5,6\}$

b.) (4 pts.) Determine the probabilities $P(A)$ and $P(B)$.

$$P(A) = \frac{n(A)}{n(\Omega)} = \frac{0}{6} = 0 \; ; \; P(B) = \frac{n(B)}{n(\Omega)} = \frac{3}{6} = \frac{1}{2}$$

c.) (4 pts.) Determine the probabilities $P(B|C)$ and $P(B \cup C)$.

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{3/6}{4/6} = \frac{3}{4}$$

$B \cup C = \{2,3,4,6\}$ so

$$P(B \cup C) = \frac{4}{6} = \frac{2}{3}$$

7.) A bag holds 6 white and 7 yellow golf balls. Randomly select 5 balls without replacement.

a.) (3 pts.) How many ways can you do this ? $C(13,5) = 1287$

b.) (5 pts.) How many ways can you select at least 1 ball of each color ?

(remove all W, all Y)

$C(13,5) - C(6,5) - C(7,5) = 1287 - 6 - 21 = 1260$

c.) (4 pts.) What is the probability that you select 3 white and 2 yellow golf balls ?

$$P(3W,2Y) = \frac{C(6,3)\cdot C(7,2)}{C(13,5)} = \frac{20\cdot 21}{1287} = \frac{420}{1287}$$

8.) (7 pts.) In the game of 5-card poker played with a standard deck of 52 cards, how many full houses (3 of one face value and 2 of another face value, for example, 3 queens and 2 sevens.) are possible ?

$$\underset{\text{triple}}{13} \cdot \underset{\text{pick 3}}{C(4,3)} \cdot \underset{\text{pair}}{12} \cdot \underset{\text{pick 2}}{C(4,2)} = (13)(4)(12)(6)$$

$$= 3744$$

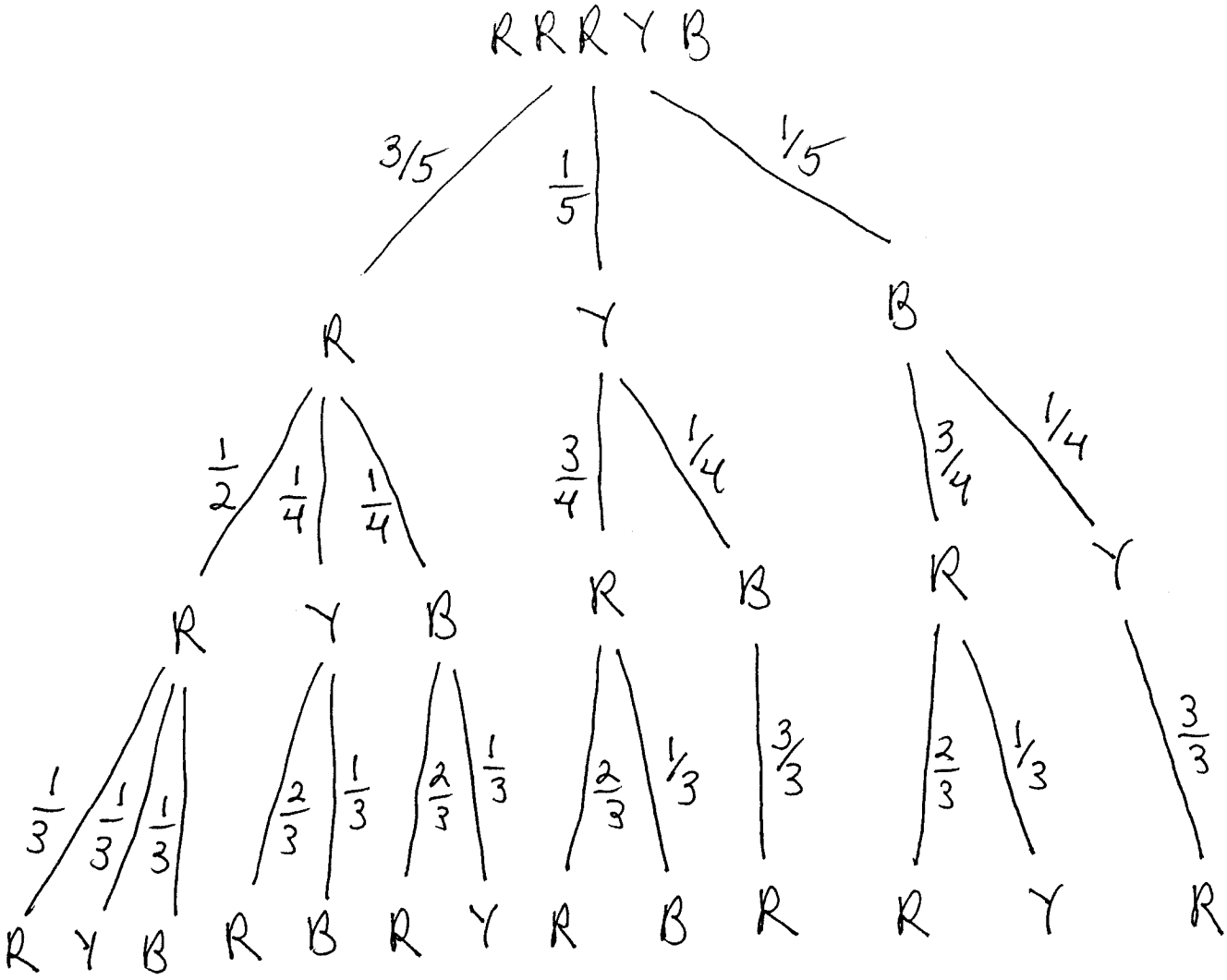
9.) (7 pts.) A committe of 6 will be chosen from a group of 12 people. How many different committes are possible if Jack and Jill cannot serve on the same committe ?

Committees with both Jack + Jill :

$\underline{J}\,\underline{J} : C(10,4)$, so remaining committees are

$C(12,6) - C(10,4) = 924 - 210$

$= 714$ committees


RRRYB
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R

10.) Consider a bag containing 3 red, 1 yellow, and 1 blue ping pong balls. Select 3 balls (one at a time) without replacement.

a.) (2 pts.) What is the probability that the first ball is yellow ?

$$P(1st\ Y) = \frac{1}{5}$$

b.) (3 pts.) What is the probability that the second ball is red given that the first ball is blue?

$$P(2nd\ R \mid 1st\ B) = \frac{3}{4}$$

c.) (5 pts.) What is the probability that the second ball is blue ?

$$P(2nd\ B) = P(RB \text{ or } YB)$$
$$= P(RB) + P(YB)$$
$$= \left(\frac{3}{5}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{4}\right)$$
$$= \frac{4}{20} = \frac{1}{5}$$

d.) (5 pts.) What is the probability that the third ball is yellow given that the first ball is red ?

$$P(3rd\ Y \mid 1st\ R)$$
$$= \frac{P(3rd\ Y \text{ and } 1st\ R)}{P(1st\ R)}$$
$$= \frac{P(RRY \text{ or } RBY)}{3/5} = \frac{5}{3}\left\{P(RRY) + P(RBY)\right\}$$
$$= \frac{5}{3}\left(\left(\frac{3}{5}\right)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + \left(\frac{3}{5}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\right)$$
$$= \frac{1}{6} + \frac{1}{12}$$
$$= \frac{3}{12} = \frac{1}{4}$$

11.) (Lotka-Voltera Predator-Prey Model) (10 pts.) Consider the system of differential equations given below, where $N(t)$ is the prey population at time t and $P(t)$ is the predator population at time t. Use separation of variables (for N and P) to solve this system of differential equations. Use the initial conditions to solve for the unknown constant C..

$$\begin{cases} \frac{dN}{dt} = 2N - PN \text{ and } N(0) = 2 \\ \frac{dP}{dt} = PN - 3P \text{ and } P(0) = 1 \end{cases} \rightarrow \frac{dN}{dP} = \frac{\frac{dN}{dt}}{\frac{dP}{dt}} = \frac{N(2-P)}{P(N-3)} \rightarrow$$

$$\int \frac{N-3}{N}\, dN = \int \frac{2-P}{P}\, dP \rightarrow$$

$$\int \left(1 - \frac{3}{N}\right) dN = \int \left(\frac{2}{P} - 1\right) dP \rightarrow N - 3\ln N = 2\ln P - P + C$$

and $N=2, P=1 \rightarrow 2 - 3\ln 2 = 2\ln 1 - 1 + C \rightarrow$

$C = 3 - 3\ln 2$ so solution is

$$N - 3\ln N = 2\ln P - P + (3 - 3\ln 2)$$

The following EXTRA CREDIT PROBLEM is worth 10 points. This problem is OPTIONAL.

1.) A bag holds 10 pink, 20 yellow, and 30 orange golf balls. In how many ways can you select 6 balls, if you must choose at least 1 ball of each color ?

$C(60,6) -$ (those missing one color)

$$= C(60,6) - \underset{PY}{C(30,6)} - \underset{PO}{C(40,6)} - \underset{YO}{C(50,6)}$$

$$+ \underset{P}{C(10,6)} + \underset{Y}{C(20,6)} + \underset{O}{C(30,6)}$$