Math 21A
Kouba
Discussion Sheet 2

1.) Consider function $f(x) = \dfrac{1}{x} + 3$. Determine a function $g(x)$ so that

a.) $f(g(x)) = x^3 + x^2$ b.) $f(g(x)) = 5f(x)$ c.) $f(g(x)) = g(x)$

2.) Write the volume of a cube as a function of its surface area. Use your result to find the volume of a cube of surface area 24 square feet.

3.) Determine the domain and range for each function.

a.) $f(x) = \ln(x^2 - 4)$ b.) $g(x) = \dfrac{e^x}{1000 + e^x}$

c.) (Challenging ... Make use of limits.) $h(x) = \dfrac{6}{3 - \sqrt{x^2 - 16}}$

d.) $f(x) = \sqrt{\dfrac{(x-1)(x-2)}{(x+3)(x+2)}}$

4.) Compute the following limits.

a.) $\lim\limits_{x \to \pi/4} \tan x$ b.) $\lim\limits_{x \to \pi/2^+} \tan x$ c.) $\lim\limits_{x \to 8} \dfrac{x^{1/3} - 2}{x - 8}$

d.) $\lim\limits_{x \to 0^-} \sin(3/x)$ e.) $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x}$ f.) $\lim\limits_{x \to -\infty} \dfrac{\sin 3x}{x}$

g.) $\lim\limits_{x \to 1^+} \dfrac{|1 - x|}{x^2 - 1}$ h.) $\lim\limits_{x \to -1} \dfrac{x^2 + 6x + 5}{x^3 + 1}$ i.) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{9x^2 + 16}}{x + 1}$

j.) $\lim\limits_{x \to -\infty} \dfrac{x^2}{x + 30}$ k.) $\lim\limits_{x \to \infty} \dfrac{x^2 - 4}{x + x^2}$ l.) $\lim\limits_{x \to \infty} \dfrac{3x^2 - 4x + 5}{6x^3 + x - 1}$

m.) $\lim\limits_{x \to \infty} \dfrac{3^x + 4^x}{2^x + 5^x}$ n.) $\lim\limits_{x \to -\infty} \dfrac{3^x + 4^x}{2^x + 5^x}$ o.) $\lim\limits_{x \to 0} \dfrac{3^x + 4^x}{2^x + 5^x}$

p.) $\lim\limits_{x \to \infty} (3^x + 5^x)^{1/x}$ q.) $\lim\limits_{x \to -\infty} (3^x + 5^x)^{1/x}$ r.) $\lim\limits_{x \to 0^+} (3^x + 5^x)^{1/x}$

s.) $\lim\limits_{x \to 0^-} (3^x + 5^x)^{1/x}$ t.) $\lim\limits_{x \to \infty} (\sqrt{x + 16} - \sqrt{x})$ u.) $\lim\limits_{x \to \infty} (\sqrt{x^2 + 4x} - \sqrt{x})$

5.) Use the Three Step Process (the definition of continuity) to determine if each of the following functions is continuous at the given $x$-value.

a.) $f(x) = \begin{cases} x^2+2, & \text{if } x > 1 \\ 4, & \text{if } x = 1 \\ \frac{6}{x+1}, & \text{if } x < 1 \end{cases}$ at $x = 1$

b.) $g(x) = \begin{cases} \frac{x^2+2x+1}{x^2+3x+2}, & \text{if } x \neq -1 \\ 0, & \text{if } x = -1 \end{cases}$ at $x = -1$

6.) For what $x$-values are the following functions continuous? Briefly explain using "short-cuts."

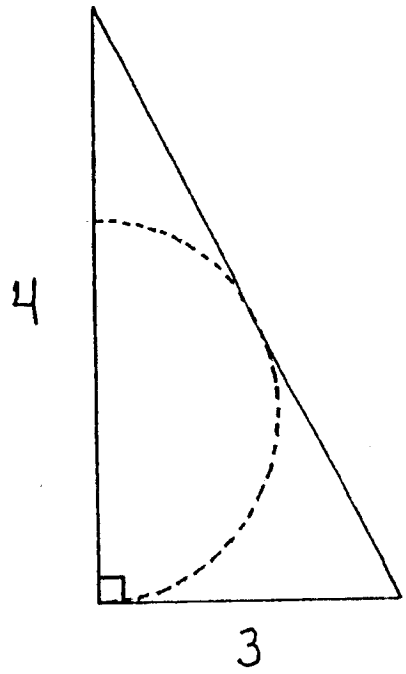
a.) $y = x^5 + x^4 + x^3 + x^2 + x + 1$ b.) $y = 7x^3 - \cos x$

c.) $y = \dfrac{x^2-4}{x^2-9}$ d.) $y = (x^2 - 3x)\sin x$ e.) $y = \sin(x^2 - 3x)$

7.) Use the Intermediate Value Theorem (IMVT) to verify that each of the following equations is solvable. This is a writing exercise as well as a math exercise. Please be organized, clear, and precise in your writing :

a.) $x^5 + x - 1 = 0$ b.) $x^3 = 10 + \sqrt{x}$ c.) $\dfrac{x^2}{x^3+4} = 1$

8.) Determine the radius of the inscribed semi-circle.

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The following problem is for recreational purposes only.

9.) Plant 10 trees in 5 straight rows of four trees each.