Math 21A
Kouba
Discussion Sheet 4

1.) Use the limit definition of derivative to compute $f'(x)$ for each of the following functions.

a.) $f(x) = \dfrac{1}{3 + \sqrt{x}}$     b.) $f(x) = \dfrac{x}{x^2 + 1}$

c.) $f(x) = \sin 3x$     d.) $f(x) = \sqrt{3 + \sqrt{x}}$

2.) Use any approved method to differentiate each of the following functions.

a.) $y = 3^{25} - \pi^{-1}$     b.) $y = 1 + 5x - 6x^5$

c.) $f(x) = x^3(x^{-5} + 3x^{2/3})$     d.) $y = \dfrac{x + 5}{x^2 - 3x - 4}$

e.) $g(x) = (x + 2)^3$     f.) $y = (5x^2 - x + 3)(4x^{-1} + x^{4/3})(1 - x + x^2 - x^3)$

3.) Determine a function whose derivative is :

a.) $f'(x) = 25x^{24} - x^{-3}$     b.) $f'(x) = 1 + 5x - 6x^5$

c.) $f'(x) = 4 - \sqrt{x}$     d.) $y' = \dfrac{x^2 + 1}{x^2}$

e.) $y' = \dfrac{(x^2 + 1) \cdot 3x^2 - x^3 \cdot 2x}{(x^2 + 1)^2}$

4.) Solve $f'(x) = 0$ for $x$. Then set up a sign chart for $f'$.

a.) $f(x) = x^4(x - 32)$     b.) $f(x) = \dfrac{x}{x^2 + 4}$     c.) $y = x - 6\sqrt{x}$

5.) Assume that $h(x) = f(x)g(x)$, $k(x) = \dfrac{f(x)}{g(x)}$, and that $f(0) = 4$, $f'(0) = 2$, $g(0) = -1$, and $g'(0) = 3$. Determine the values of $h'(0)$ and $k'(0)$. .

6.) Use the limit definition of derivative to show that $f(x) = |x|$ is NOT differentiable at $x = 0$, i.e., show that $f'(0)$ does not exist.

7.) Use the limit definition of derivative to show that the following function IS differentiable at $x = 1$, i.e., show that $f'(1)$ does exist.
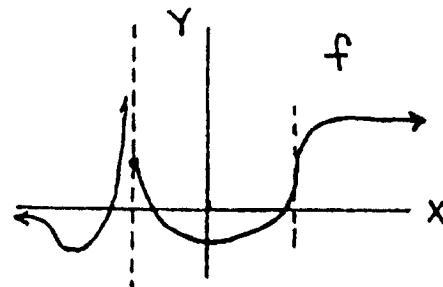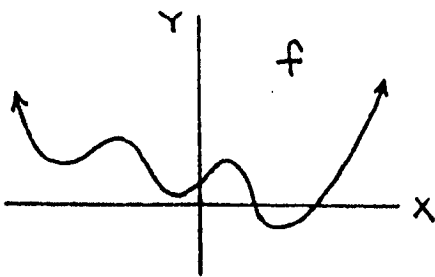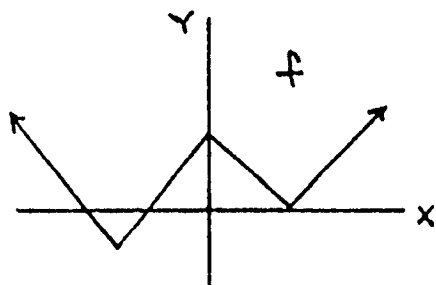
$$f(x) = \begin{cases} 2 + \sqrt{x}, & \text{if } x \geq 1 \\ \dfrac{1}{2}x + \dfrac{5}{2}, & \text{if } x < 1 \end{cases}$$

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8.) Draw a possible graph of the derivative $f'$ using the given graph of the function $f$.
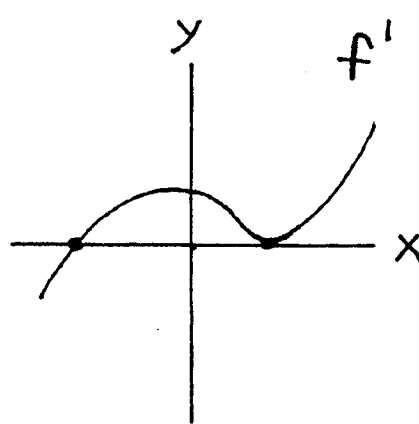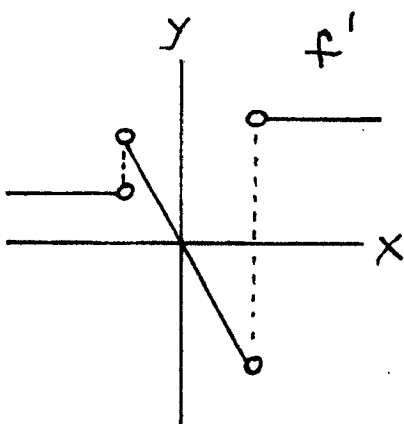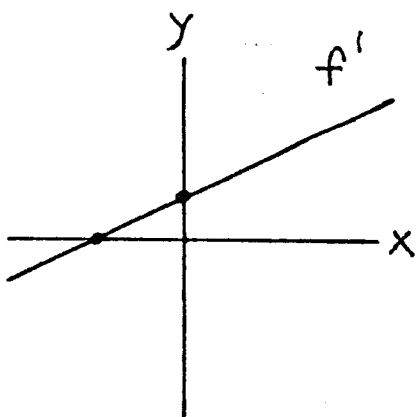
a.)                 b.)                 c.)

9.) Draw a possible graph for the function $f$ using the given graph of the derivative $f'$.

a.)                 b.)                 c.)

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The following problem is for recreational purposes only.

10.) A snail is at the bottom of a well which is 100 feet deep. Each day it climbs up 5 feet and back down 4 feet. In how many days will the hapless snail reach the top of the well ?