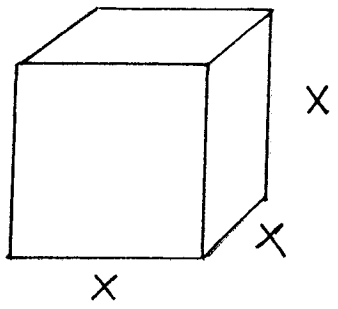
Math 21A
Kouba
Challenge Discussion Sheet 2



1.) Write the surface area of a cube with edge length x as a function of the volume of the cube.

2.) Determine the following limits.

a.) $\lim_{x \to 0} x^4 \sin(1/x)$ b.) $\lim_{x \to 0^-} \dfrac{x + \sin x}{\sqrt{x^2 + 9x}}$

c.) $\lim_{x \to -\infty} \dfrac{x^3 - x^2 + x - 1}{1 - x + x^2 - x^3}$ d.) $\lim_{x \to \infty} \arcsin\left(\dfrac{x^2 + 4x}{2x^2 + 4}\right)$

e.) $\lim_{x \to \infty} \left(\sqrt{x^2 + 4x} - \sqrt{x^2 - 4}\right)$ f.) $\lim_{x \to -\infty} \left(2x - \sqrt{x^2 + 16}\right)$

3.) Let $f(x) = \begin{cases} \dfrac{x^3 + x^2 - 2x}{x^2 - 2x}, & \text{if } x \neq 0, x \neq 2 \\ 1, & \text{if } x = 0 \\ -3, & \text{if } x = 2. \end{cases}$

i.) Show that f is continuous at $x = 0$.

ii.) Show that f is not continuous at $x = 2$.

iii.) Is f continuous for all values of x not equal to 0 and not equal to 2 ? Why ?

iv.) Sketch the graph of f.

4.) For what values of x is each of the following functions continuous ? Briefly explain.

a.) $y = (x^2 + x + 1)^{100}$

b.) $f(x) = (\ln x)^4 + \sin x$

5.) Use the Intermediate Value Theorem to prove that

a.) $x^5 = x^2 + 1$ has at least one solution.

b.) $x^4 = 3^x$ has at least three solutions.