Math 21A
Kouba
Challenge Discussion Sheet 9

1.) Evaluate the following limits by using one of the three limit definitions for e.

a.) $\lim\limits_{n \to \infty} \left(1 + \dfrac{4}{n}\right)^{3n}$

b.) $\lim\limits_{n \to \infty} \left(1 + \dfrac{3}{7n}\right)^{n}$

c.) $\lim\limits_{n \to -\infty} \left(1 - \dfrac{2}{3n}\right)^{-4n}$

d.) $\lim\limits_{n \to \infty} \left(\dfrac{n+3}{n-2}\right)^{n}$

e.) $\lim\limits_{h \to 0} \left(1 - \dfrac{h}{2}\right)^{3/h}$

2.) Solve for x.

a.) $\ln(2x+3) - \ln(x+4) = 0$

b.) $\log(x) + \log(x-21) = 2$

c.) $e^{3x} + 2 \cdot e^{x} = 3 \cdot e^{2x}$

d.) $e^{3x} - 3 \cdot e^{2x} - 2 \cdot e^{x} + 6 = 0$

3.) Solve $f'(x) = 0$ and $f''(x) = 0$ for x and set up a sign chart for each.

a.) $f(x) = x \ln x$.

b.) $f(x) = xe^{-x^2/2}$.

4.) Find $y' = \dfrac{dy}{dx}$. Do not simplify your answers.

a.) $y = \ln[x(x+1)(x+2)(x+3)(x+4)]$

b.) $y = (x+1)^{e^{\sin x}}$

c.) $x^x \cdot y^3 = x^{\ln y}$

5.) Consider the rectangle in the given sketch inscribed in the first quadrant below the graph of $y = e^{(-3/2)x}$. Determine the dimensions of the rectangle

a.) with maximum area.
b.) with minimum perimeter.
c.) with minimum diagonal. Use Newton's method to estimate the root of the derivative equation to three decimal places.