

Math 21A  
Kouba  
Discussion Sheet 4

1.) Use the limit definition of derivative to compute  $f'(x)$  for each of the following functions.

a.)  $f(x) = \frac{1}{3 + \sqrt{x}}$

b.)  $f(x) = \frac{x}{x^2 + 1}$

c.)  $f(x) = \sin 3x$

d.)  $f(x) = \sqrt{3 + \sqrt{x}}$

2.) Use any method to differentiate each of the following functions.

a.)  $y = 1 + 5x - 6x^5$

b.)  $f(x) = x^3 \sin x$

c.)  $y = \frac{x + 5}{x^2 \tan x}$

d.)  $g(x) = \frac{x \cos x}{\tan x - 5 \sec x}$

3.) Determine a function whose derivative is :

a.)  $f'(x) = 1 + 5x - 6x^5$

b.)  $f'(x) = 4 - \sqrt{x}$

c.)  $y' = \frac{x^2 + 1}{x^2}$

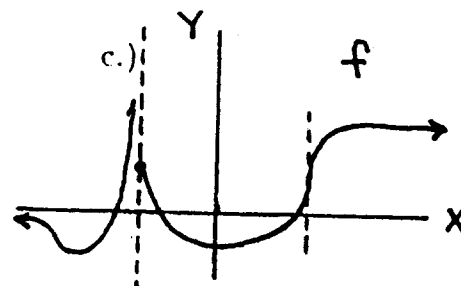
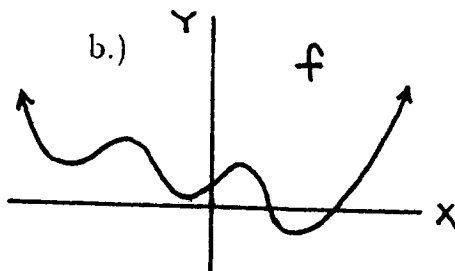
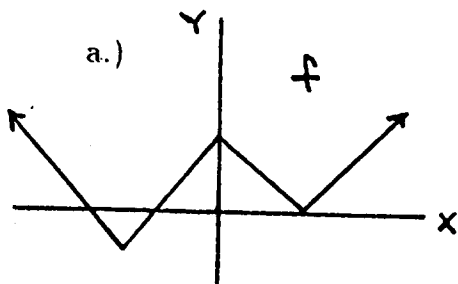
d.)  $y' = \frac{4x^3 + 3x^2 + 2x + 1}{x^4 + x^3 + x^2 + x + 1}$

4.) Use the limit definition of derivative to show that  $f(x) = |x|$  is NOT differentiable at  $x = 0$ , i.e., show that  $f'(0)$  does not exist.

5.) Use the limit definition of derivative to show that the following function IS differentiable at  $x = 1$ , i.e., show that  $f'(1)$  does exist.

$$f(x) = \begin{cases} 2 + \sqrt{x}, & \text{if } x \geq 1 \\ \frac{1}{2}x + \frac{5}{2}, & \text{if } x < 1 \end{cases}$$

6.) Draw a possible graph for  $f'$  using the given graph of  $y = f(x)$ .



7.) Let  $f(x) = \frac{x}{x^2 + 1}$ . Solve  $f'(x) = 0$  for  $x$ . What is the geometric significance of these  $x$ -values?

8.) Assume that  $h(x) = f(x)g(x)$  and that  $f(0) = 1$ ,  $f'(0) = 2$ ,  $g(0) = -1$ , and  $g'(0) = 3$ . Determine the value of  $h'(0)$ .

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The following problem is for recreational purposes only.

9.) A snail is at the bottom of a well which is 100 feet deep. Each day it climbs up 5 feet and back down 4 feet. In how many days will the hapless snail reach the top of the well?