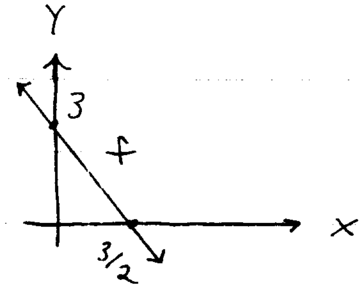
HW #1

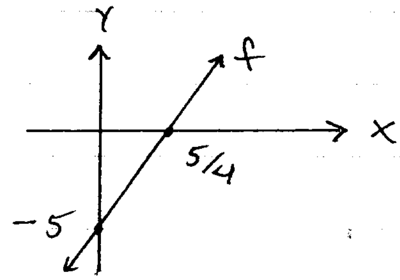
-13

Section 2.1

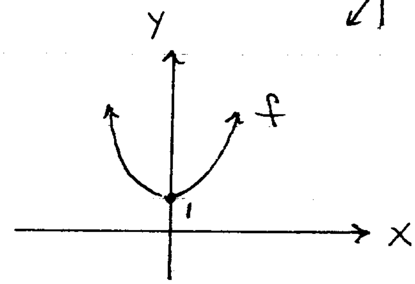
$\boxed{20:3}$ $f(x) = -2x+3$



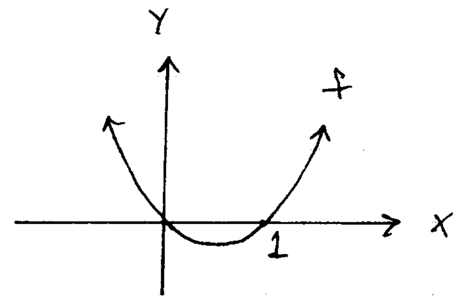
$\boxed{20:4}$ $f(x) = 4x-5$



$\boxed{20:5}$ $f(x) = 3x^2+1$



$\boxed{20:9}$ $f(x) = x^2 - x$
$= x(x-1)$

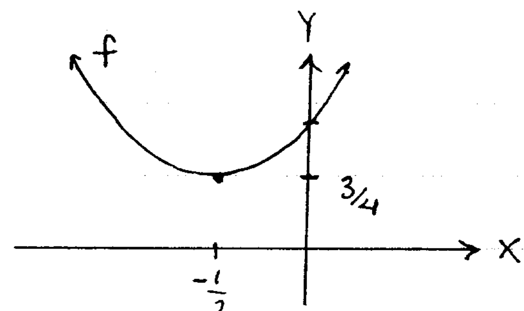


$\boxed{20:11}$ $f(x) = x^2+x+1$

complete the square →

$= \left(x^2+x+\frac{1}{4}\right) + \frac{3}{4}$

$= \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$

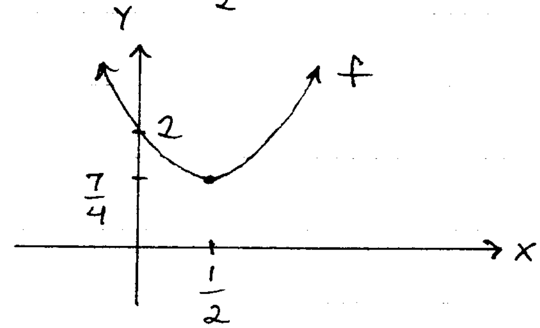


$\boxed{20:12}$ $f(x) = x^2-x+2$

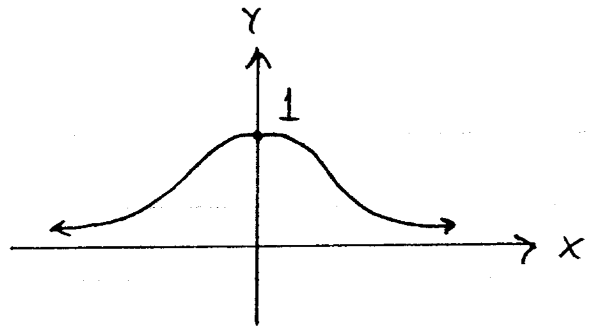
complete the square →

$= \left(x^2-x+\frac{1}{4}\right) + \frac{7}{4}$

$= \left(x-\frac{1}{2}\right)^2 + \frac{7}{4}$



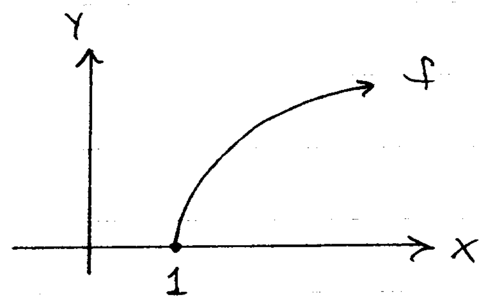
$\boxed{20:14}$ $f(x) = \dfrac{1}{2x^2+1}$



$\boxed{20:20}$ $f(x) = \sqrt{x-1}$

domain : all $x \geq 1$

range : all $Y \geq 0$



$\boxed{20:22}$ $f(x) = \sqrt{x^2-4} = \sqrt{(x-2)(x+2)}$

+ 0 − 0 + value of $(x-2)(x+2)$,

$x=-2$ $x=2$

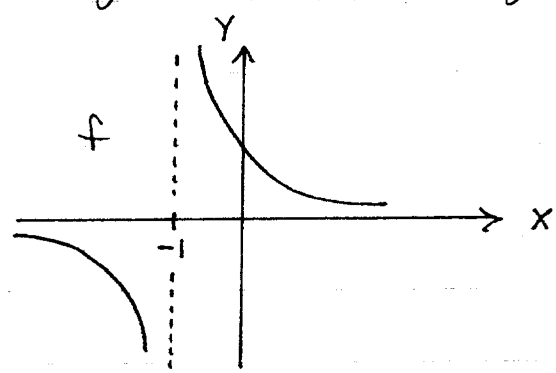
domain : $x \geq 2$, $x \leq -2$

range : all $Y \geq 0$ (why?)

$\boxed{20:25}$ $f(x) = \dfrac{1}{x+1}$

domain : $x < -1$, $x > -1$

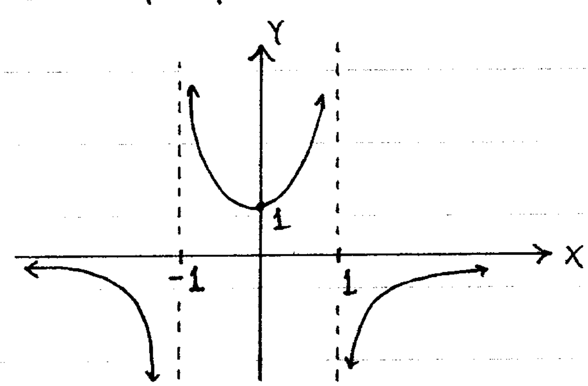
range : $Y > 0$, $Y < 0$



$\boxed{20:27}$ $f(x) = \dfrac{1}{1-x^2} = \dfrac{1}{(1-x)(1+x)}$

domain : $x < -1$, $-1 < x < 1$, $x > 1$

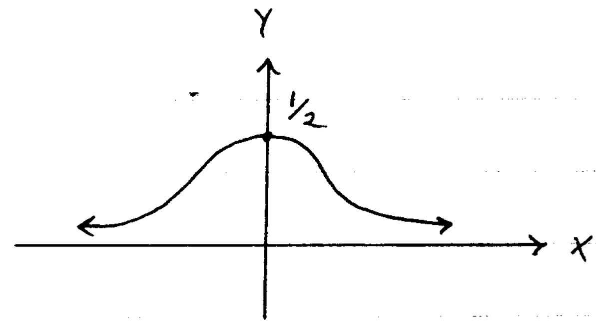
range : $Y \geq 1$, $Y < 0$



$\boxed{20:28}$ $f(x) = \dfrac{1}{2+x^2}$

domain : all x-values

range : $0 < Y \leq 1/2$



$\boxed{20:30}$ $f(x) = \dfrac{1}{1+x}$ a.) $f(-3) = \dfrac{1}{-2} = -0.5$

b.) $f(3) = \frac{1}{4} = 0.25$ c.) $f(9) = \frac{1}{10} = 0.1$

d.) $f(99) = \frac{1}{100} = 0.01$

$\boxed{20:33}$ $f(x) = x^2$ so $\dfrac{f(3+h)-f(3)}{h} = \dfrac{(3+h)^2 - 3^2}{h}$

$$= \frac{\cancel{9}+6h+h^2-\cancel{9}}{h} = \frac{\cancel{h}(6+h)}{\cancel{h}} = 6+h$$

h	$6+h$
1	7
0.01	6.01
-0.01	5.99
0.0001	6.0001

as h gets smaller, $\dfrac{f(3+h)-f(3)}{h} = 6+h$ gets closer to 6 .

$\boxed{20:38}$ $f(x) = \dfrac{1}{2x+1}$ so $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{1}{2(x+h)+1} - \frac{1}{2x+1}}{\frac{h}{1}}$

$$= \left(\frac{1}{2x+2h+1} - \frac{1}{2x+1}\right)\cdot\frac{1}{h} = \frac{\cancel{2x}+\cancel{1}-\cancel{2x}-2h-\cancel{1}}{(2x+2h+1)(2x+1)\,h} = \frac{-2\cancel{h}}{(2x+2h+1)(2x+1)\cancel{h}}$$

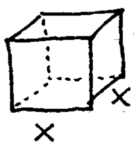
$$= \frac{-2}{(2x+2h+1)(2x+1)}$$

$\boxed{20:39}$ a.) not function

b.) function

c.) function

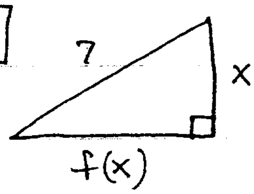
$\boxed{20:44}$



volume $f(x) = x^3$

$\boxed{20:45}$ surface area $f(x) = 6x^2$

$\boxed{20:47}$

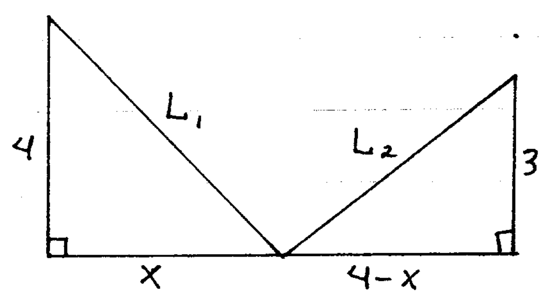


$(f(x))^2 + x^2 = 7^2$ so

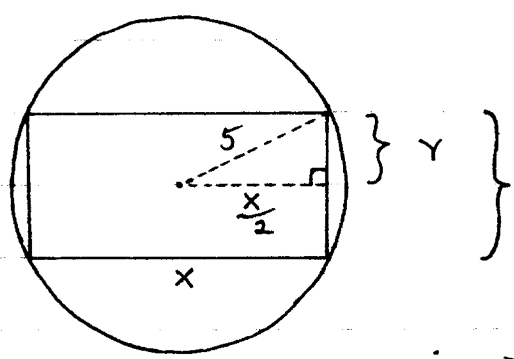
$f(x) = \sqrt{49 - x^2}$

$\boxed{20:48}$ $f(x) = L_1 + L_2$

$= \sqrt{16 + x^2} + \sqrt{(4-x)^2 + 9}$



$\boxed{20:49}$



$\left(\frac{x}{2}\right)^2 + Y^2 = 5^2 \rightarrow$

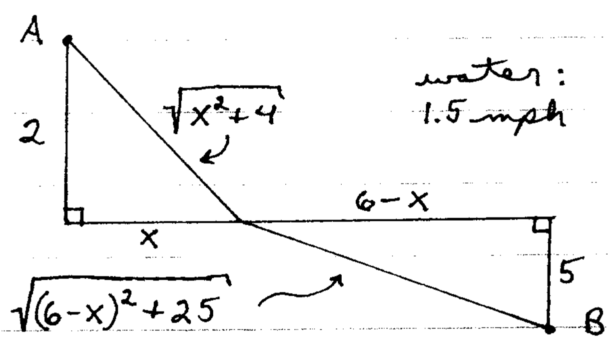
$Y = \sqrt{25 - \frac{x^2}{4}}$ so

perimeter of rectangle is

$f(x) = 2x + 2(2Y) = 2x + 4Y = 2x + 4\sqrt{25 - \frac{x^2}{4}}$.

$\boxed{20:50}$ time $= \dfrac{\text{distance}}{\text{rate}}$,

so total time

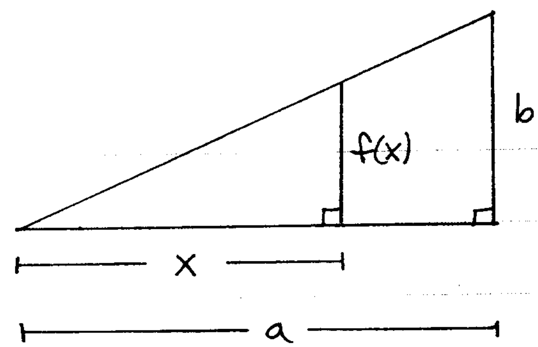


water: 1.5 mph

$f(x) = (\text{time swim}) + (\text{time walk})$

$= \dfrac{\sqrt{x^2+4}}{1.5} + \dfrac{\sqrt{(6-x)^2+25}}{4}$.

land: 4 mph



$\boxed{20:52}$ a.) $f(0)=0$, $f(a)=b$

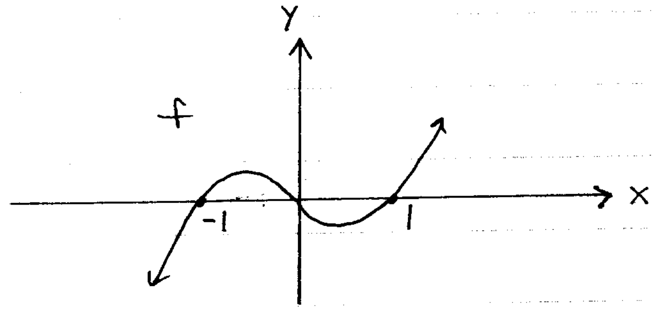
b.) and c.) by similar triangles

$$\frac{f(x)}{x} = \frac{b}{a} \longrightarrow f(x) = \frac{b}{a}x \quad \text{so } f\left(\frac{a}{2}\right) = \frac{b}{a}\cdot\frac{a}{2} = \frac{b}{2}.$$

$\boxed{20:56}$ $f(x) = x(x+1)(x-1)$

a.) $f(x)=0$ for $x=-1,0,1$

b.) See a.) c.) $Y=0$



$\boxed{20:58}$ a.) $f(x)=x^2$ so $f(a+b) = (a+b)^2 = a^2+2ab+b^2$
$= f(a)+2ab+f(b) \neq f(a)+f(b)$. NO

b.) $f(x)=3x$ so $f(a+b)=3(a+b)=3a+3b = f(a)+f(b)$. YES

c.) $f(x)=-4x$ so $f(a+b)=-4(a+b)=-4a+{}^-4b = f(a)+f(b)$. YES

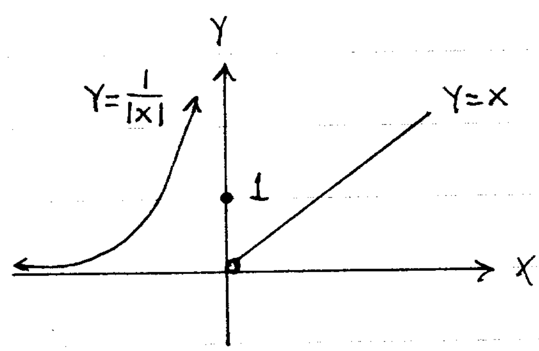
d.) $f(x)=\sqrt{x}$ so $f(a+b)=\sqrt{a+b} \neq \sqrt{a}+\sqrt{b} = f(a)+f(b)$. NO

e.) $f(x)=2x+1$ so $f(a+b)=2(a+b)+1 = 2a+2b+1$
$= (2a+1)+(2b+1)-1 = f(a)+f(b)-1$
$\neq f(a)+f(b)$. NO

$\boxed{20:60}$ $f(-x) = \frac{1}{f(x)} \rightarrow f(x)f(-x)=1$:

1) $f(x)=1$ works

2.) $f(x)=-1$ works



3.) $f(x) = \begin{cases} x & \text{for } x>0 \\ 1 & \text{for } x=0 \\ \frac{1}{|x|} & \text{for } x<0 \end{cases}$ works.

Section 2.2

$\boxed{26:6}$ $2 \to \frac{1}{3}$ power $\to$ cos $\to$ square

$\boxed{26:17}$ $y = \cos(1 + \tan^2 x)$

$\boxed{26:25}$ Let $y = u^3$, $u = \cos v$, $v = 2x$, then $y = \cos^3 2x$.

$\boxed{26:28}$ (a) $g(0.6) \approx 0.2$ and $f(g(0.6)) \approx f(0.2) \approx 0.3$

(b) $f(0.3) \approx 0.5$ and $g(f(0.3)) \approx g(0.5) \approx 0.3$

(c) $f(0.5) \approx 0.7$ and $f(f(0.5)) \approx f(0.7) \approx 0.8$

$\boxed{26:29}$ $f(x) = 2x^2 - 1$, $g(x) = 4x^3 - 3x$

(a) $(f \circ g)(x) = f(g(x)) = f(4x^3 - 3x)$

$= 2(4x^3 - 3x)^2 - 1 = 2[16x^6 - 24x^4 + 9x^2] - 1$

$= 32x^6 - 48x^4 + 18x^2 - 1$

(b) $(g \circ f)(x) = g(f(x)) = g(2x^2 - 1)$

$= 4(2x^2 - 1)^3 - 3(2x^2 - 1) = 4(8x^6 - 12x^4 + 6x^2 - 1) - 3(2x^2 - 1)$

$= 32x^6 - 48x^4 + 18x^2 - 1$

$\boxed{26:30}$ $f(x) = \frac{1}{1-x}$; domain of f is all $x \neq 1$;

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{1-x}\right) = \frac{1}{1 - \frac{1}{1-x}} = \frac{1}{\frac{1-x-1}{1-x}}$$

$= \frac{1-x}{-x} = \frac{x-1}{x}$; domain of $f \circ f$ is all $x \neq 0$ and $x \neq 1$;

$$(f \circ f \circ f)(x) = f(f(f(x))) = f\left(\frac{x-1}{x}\right) = \frac{1}{1 - \frac{x-1}{x}}$$

$= \dfrac{1}{1-1+\frac{1}{x}} = x$; domain is all $x \neq 0$ and $x \neq 1$.

$\boxed{26:31}$ $g(x) = x^2$, $f(x) = ax+b$ then

$f(g(x)) = f(x^2) = ax^2+b$ and

$g(f(x)) = g(ax+b) = (ax+b)^2 = a^2x^2+2abx+b^2$; if

$f(g(x)) = g(f(x))$ then

$$ax^2+b = a^2x^2+2abx+b^2 \rightarrow$$

$$(a-a^2)x^2 - 2abx + (b-b^2) = 0 \rightarrow$$

$a-a^2 = a(1-a) = 0$ and $-2ab = 0$ and $b-b^2 = b(1-b) = 0$;

then $a(1-a) = 0$ means $a=0$ or $a=1$; but we assume $a \neq 0$ so $\boxed{a=1}$; since $-2ab = 0$, then $\boxed{b=0}$

so $f(x) = ax+b = 1\cdot x + 0 = x$.

$\boxed{26:33}$ $f(x) = 2x+3$, $g(x) = ax+b$ then

$f(g(x)) = f(ax+b) = 2(ax+b)+3 = 2ax+(2b+3)$ and

$g(f(x)) = g(2x+3) = a(2x+3)+b = 2ax+(3a+b)$;

if $f(g(x)) = g(f(x))$ then

$$\cancel{2ax}+(2b+3) = \cancel{2ax}+3a+b \rightarrow$$

$$b+3 = 3a \rightarrow b = 3a-3 \text{ so}$$

$g(x) = ax+b = ax+(3a-3)$ where a can be any real number.

$\boxed{26:35}$ $f(x) = x^5$ and $f(g(x)) = x$ then

$(g(x))^5 = x \rightarrow g(x) = x^{1/5}$.