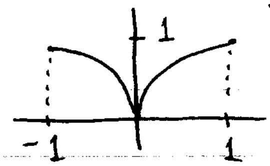
HW #12

-24

Section 4.1

$\boxed{172:5}$ a.)

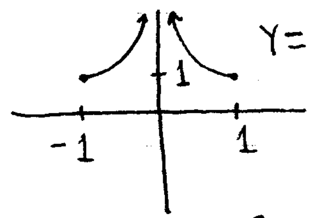


b.) $f(1) = 1 = f(-1)$

c.) $f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}} = 0$. (NOT possible)

d.) This example does not contradict Rolle's Theorem since f is not differentiable on $(-1,1)$ ($f'(0)$ does not exist)

$\boxed{172:6}$ a.)



b.) $f(1) = 1 = f(-1)$

c.) $f'(x) = -2x^{-3} = \frac{-2}{x^3} = 0$ (NOT possible)

d.) This example does not contradict Rolle's Theorem since f is not continuous on $[-1,1]$ (f is not defined at $x=0$)

$\boxed{172:8}$ $f(x) = x^3 - x$ is continuous on $[-1,1]$ and $f(1) = 0 = f(-1)$ since f is a polynomial, and $f'(x) = 3x^2 - 1$ so f is differentiable on $(-1,1)$ so Rolle's Theorem applies; $f'(c) = 0 \rightarrow 3c^2 - 1 = 0 \rightarrow c = \pm\frac{1}{\sqrt{3}}$

$\boxed{172:10}$ $f(x) = \sin x + \cos x$ is continuous on $[0,4\pi]$ and $f(0) = 1 = f(4\pi)$ since f is the sum of continuous functions, and $f'(x) = \cos x - \sin x$ so f is differentiable on $(0,4\pi)$ and Rolle's Theorem applies;

$f'(c) = 0 \rightarrow \cos c - \sin c = 0 \rightarrow \cos c = \sin c$
$\rightarrow c = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$.

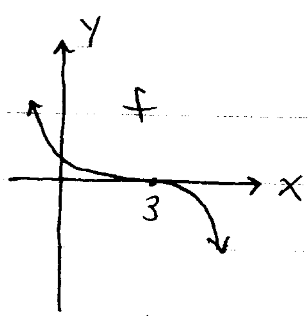
$\boxed{172:12}$ $f(x) = 2x^2 + x + 1$ is continuous on $[-2,3]$ since f is a polynomial, and $f'(x) = 4x+1$ so f is differentiable on $(-2,3)$. Then by MVT there is at least one c in $(-2,3)$ satisfying $f'(c) = \dfrac{f(3) - f(-2)}{3-(-2)}$

$\rightarrow 4c+1 = \dfrac{22-7}{5} \rightarrow 4c+1 = 3 \rightarrow c = \frac{1}{2}$.

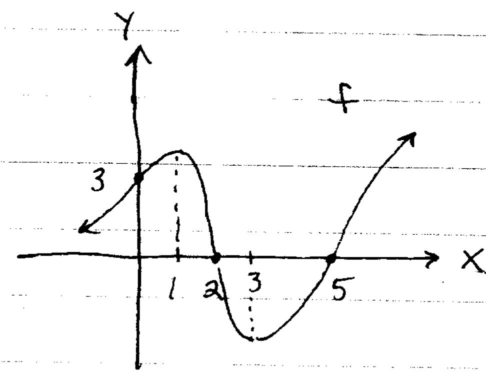
$\boxed{172:17}$ a.) $D \sec^2 x = 2\sec x \cdot \sec x \tan x$
$D \tan^2 x = 2\tan x \cdot \sec^2 x$

b.) $\sec^2 x = \tan^2 x + C$ (let $x=0$) $\rightarrow$
$\sec^2 0 = \tan^2 0 + C \rightarrow 1 = 0 + C \rightarrow C = 1 \rightarrow$
$\sec^2 x = \tan^2 x + 1$

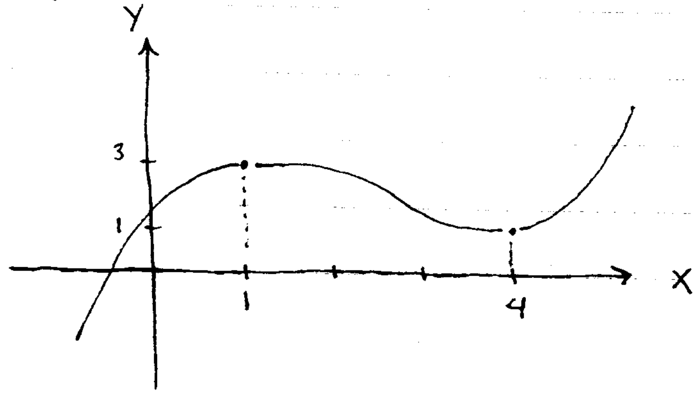
$\boxed{172:20}$



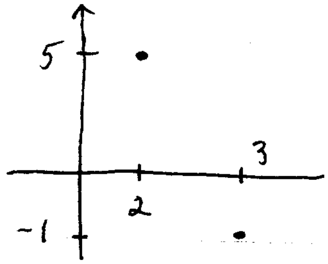
$\boxed{172:22}$



$\boxed{172:24}$



$\boxed{172:26}$



$\boxed{\text{If}}$ $f(2)=5$, $f(3)=-1$, and $f'(x)\geq 0$ for all x, then f is continuous for all x. In particular, f is continuous on $[2,3]$ and differentiable on $(2,3)$. So by MVT there is at least one c in $(2,3)$ satisfying

$$f'(c)=\frac{f(3)-f(2)}{3-2} \rightarrow f'(c)=\frac{-1-5}{1}=-6$$

$\rightarrow f'(c)=-6$. But this is $\boxed{\text{impossible}}$ since $f'(x)\geq 0$ for all x.

$\boxed{172:37}$ $f(x)=7x+k\sin 2x \rightarrow$

$$f'(x)=7+k\cdot\cos 2x\cdot 2 = 7+2k\cos 2x$$
$$=7\left(1+\tfrac{2}{7}k\cdot\cos 2x\right)$$

; since $-1\leq \cos 2x\leq 1$ then $-1<\frac{2}{7}k<1$ will guarantee that $1+\frac{2}{7}k\cdot\cos 2x>0$, i.e., $f'(x)>0$, i.e., f is increasing. Choose $-\frac{7}{2}<k<\frac{7}{2}$.

$\boxed{172:38}$ Assume $f(x)=x^3+ax^2+c$ with $a<0$ and $c>0$. Show f has exactly one negative root:

$$f'(x)=3x^2+2ax = 3x\left(x+\tfrac{2}{3}a\right)=0$$

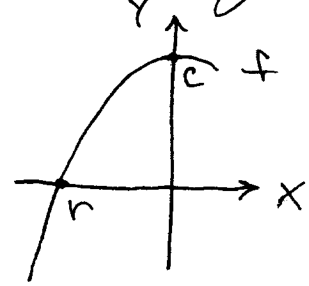
$\swarrow$ $\downarrow$

$x=0$ $\quad$ $x=-\frac{2}{3}a>0$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & x=0 & & x=-\frac{2}{3}a & \end{array} \quad f'$$

Since $f(0)=c>0$ and f is increasing for $x<0$ and $\lim\limits_{x\to-\infty} f(x)$

$= \lim\limits_{x\to-\infty} \left(x^2(x+a)+c\right) = -\infty$,



it follows that f has exactly one negative root r.

$\boxed{172:43}$ $f(t) = -16t^2 + 32t + 40$ (continuous, differentiable)

a.) $f(0) = 40$ ft., $f(2) = 40$ ft.

b.) Rolle's: $f'(c) = 0$, i.e., instantaneous velocity is zero at time $t=c$.

c.) $f'(t) = -32t + 32$ so $f'(c)=0 \rightarrow$

$-32c + 32 = 0 \rightarrow c = 1$ sec.

$\boxed{172:46}$ Let $f(x) = x^3$, then f is increasing for all values of x but $f'(x)$ is not always positive, since $f'(0) = 0$.