HW #13

Section 4.3

$\boxed{190:5}$ $Y = 2x^3 + x + 2 \rightarrow Y' = 6x^2 + 1 \rightarrow Y'' = 12x$

$\boxed{190:7}$ $Y = \dfrac{x}{x+1} \rightarrow Y' = \dfrac{(x+1)(1) - x(1)}{(x+1)^2} = (x+1)^{-2} \rightarrow$

$Y'' = -2(x+1)^{-3} = \dfrac{-2}{(x+1)^3}$

$\boxed{190:9}$ $Y = x\cos x \rightarrow Y' = x \cdot {}^{-}\sin x + \cos x = \cos x - x\sin x$

$\rightarrow Y'' = -\sin x - (x\cos x + \sin x) = -2\sin x - x\cos x$

$\boxed{190:10}$ $Y = \sec x \rightarrow Y' = \sec x \tan x \rightarrow$

$Y'' = \sec x \cdot \sec^2 x + \sec x \cdot \tan x \cdot \tan x$

$= \sec^3 x + \sec x \cdot \tan^2 x = \sec^3 x + \sec x (\sec^2 x - 1)$

$= 2\sec^3 x - \sec x$

$\boxed{190:12}$ $Y = \dfrac{x}{\tan x} \rightarrow Y' = \dfrac{\tan x - x \cdot \sec^2 x}{\tan^2 x}$

$= \dfrac{\tan x}{\tan^2 x} - \dfrac{x\sec^2 x}{\tan^2 x} = \cot x - x\csc^2 x \rightarrow$

$Y'' = -\csc^2 x - [x \cdot 2\csc x \cdot {}^{-}\csc x \cot x + \csc^2 x]$

$= -\csc^2 x + 2x\csc^2 x \cot x - \csc^2 x$

$= -2\csc^2 x + 2x\csc^2 x \cot x$

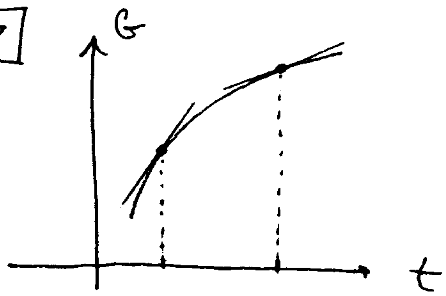
$\boxed{190:15}$ $Y = \sin 3x \rightarrow Y' = 3\cos 3x \rightarrow Y'' = -9\sin 3x$

$\boxed{190:16}$ $Y = \tan x^2 \rightarrow Y' = \sec^2 x^2 \cdot 2x = 2x \cdot \sec^2 x^2 \rightarrow$

$Y'' = 2x \cdot 2\sec x^2 \cdot \sec x^2 \tan x^2 \cdot 2x + 2\sec^2 x^2$

$= 8x^2 \sec^2 x^2 \cdot \tan x^2 + 2\sec^2 x^2$

$\boxed{190:18}$



The price goes higher but the slope (derivative) is smaller.

$\boxed{190:23}$ Let $s(t)$ be distance (miles) from airport at time t (hours.);

let A be acceleration of plane,

let T be ~~total~~ time ~~to~~ fly 120 miles,

then $s(0) = 120$ mi., $s'(0) = -500$ mph $\}$ $s(t)$ is

$s(T) = 0$ mi., $s'(T) = -180$ mph. $\}$ decreasing!

Thus, acceleration

$s''(t) = A \rightarrow$ velocity $s'(t) = At + c$

(and $s'(0) = -500 \rightarrow -500 = A(0) + c \rightarrow c = -500$) so

$\boxed{s'(t) = At - 500}$ and distance $s(t) = \frac{A}{2}t^2 - 500t + c$

(and $s(0) = 120 \rightarrow 120 = \frac{A}{2}(0)^2 - 500(0) + c \rightarrow c = 120$) so

$\boxed{s(t) = \frac{A}{2}t^2 - 500t + 120}$.

Now

$s'(T) = -180 \rightarrow AT - 500 = -180 \rightarrow \boxed{AT = 320}$ and

$s(T) = 0 \rightarrow \boxed{\frac{A}{2}T^2 - 500T + 120 = 0}$; $A = \frac{320}{T} \rightarrow$

$\frac{1}{2} \cdot \frac{320}{T} T^2 - 500T + 120 = 0 \rightarrow -340T = -120 \rightarrow T \simeq .35$ hr.

and $A \simeq 907$ mph./hr.

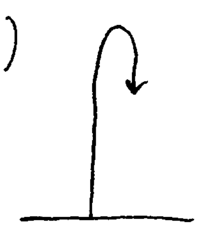
$\boxed{190:26}$ a.) $s(0) = 10$ ft., $s'(0) = 8$ ft./sec., $s''(t) = -8$ ft./sec.^{2}

$\rightarrow s'(t) = -8t + c$ $(s'(0) = 8 \rightarrow c = 8)$ so

$s'(t) = -8t + 8 \rightarrow s(t) = -4t^2 + 8t + c$ $(s(0) = 10 \rightarrow$

$c = 10)$ so $s(t) = -4t^2 + 8t + 10$.

b.)



$s(t)$ is largest when $s'(t) = 0 \rightarrow$

$-8t + 8 = 0 \rightarrow t = 1$ sec. $\rightarrow$ max. ht. is

$s(1) = 14$ ft.

$\boxed{190:29}$ Let $s(t)$ be distance traveled (**feet**) in t seconds. Then

$s'(0) = 0$ ft./sec., $s'(15) = 88$ ft./sec.,
$s(0) = 0$ ft.; let A be constant accel.

so $s''(t) = A \rightarrow s'(t) = At + c$ ($s'(0) = 0 \rightarrow c = 0$ and $s'(15) = 88 \rightarrow 88 = A(15) \rightarrow A = \frac{88}{15}$ ft./sec.^{2}) so

$$s'(t) = \frac{88}{15}t \rightarrow s(t) = \frac{44}{15}t^2 + c \;\; (s(0) = 0 \rightarrow c = 0)$$

and $s(t) = \frac{44}{15}t^2 \rightarrow$ $s(15) = 660$ ft.

$\boxed{190:30}$ Assume a ball is thrown upward at V_0 ft./sec., then

$$s'' = -32 \rightarrow$$
$$s' = -32t + c \;\; (t=0,\ s' = V_0) \rightarrow c = V_0 \rightarrow$$
$$\boxed{s' = -32t + V_0} \rightarrow$$
$$s = -16t^2 + V_0 t + c \;\; (t=0,\ s=0) \rightarrow c = 0 \rightarrow$$
$$\boxed{s = -16t^2 + V_0 t};$$

highest point: $s' = 0 \rightarrow -32t + V_0 = 0 \rightarrow$
$t = V_0/32$ seconds;

strike ground: $s = 0 \rightarrow -16t^2 + V_0 t = 0 \rightarrow$
$t(-16t + V_0) = 0 \rightarrow -16t + V_0 = 0 \rightarrow t = \frac{V_0}{16}$ sec.;

$t = 0$: $s' = V_0$, speed $= |V_0| = V_0$
$t = \frac{V_0}{16}$: $s' = -V_0$, speed $= |-V_0| = V_0$.

Math 21A
Kouba
Worksheet 3

1.) A ball is thrown straight up from the top of a 128-ft. high building at 24 ft./sec.

a.) In how many seconds will the ball reach its highest point ? How high is the ball above the ground at this point ?
b.) In how many seconds will the ball strike the ground ?
c.) What is the ball's velocity as it strikes the ground ?

2.) A ball is thrown straight down from the top of a 128-ft. high building at 32 ft./sec.

a.) In how many seconds will the ball strike the ground ?
b.) What is the ball's velocity as it strikes the ground ?

3.) A ball is thrown horizontally from the top of a 128-ft. high building at 100 ft./sec.

a.) In how many seconds will the ball strike the ground ?
b.) What is the ball's velocity as it strikes the ground ?
c.) How far from the base of the building will the ball hit the ground ?

4.) A car increases speed with constant acceleration from 0 miles per hour to 60 miles per hour in 15 seconds. How long does it take the car to go from 60 miles per hour to 100 miles per hour at the same constant acceleration ? What is the car's acceleration ? How far does the car travel as it goes from 0 and 100 miles per hour ?

5.) A car traveling at 40 ft./sec. applies the brakes and begins a constant deceleration. If the car comes to rest in 100 feet after it applies the brakes, how long does it take to come to a complete stop ? What is the car's deceleration ?

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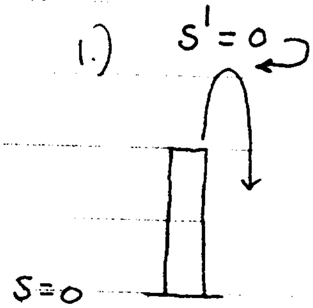
Worksheet 3 Solutions

1.)



$s'' = -32 \rightarrow$

$s' = -32t + c \quad (t=0,\ s' = 24)$

$24 = 0 + c \rightarrow c = 24$

$\boxed{s' = -32t + 24} \rightarrow$

$s = -16t^2 + 24t + c \quad (t=0,\ s = 128)$

$128 = 0 + c \rightarrow c = 128$

$\boxed{s = -16t^2 + 24t + 128}$;

a.) highest point : $s' = 0 \rightarrow -32t + 24 = 0 \rightarrow$

$t = \frac{24}{32} = \frac{3}{4}$ sec. and $s = 137$ ft.

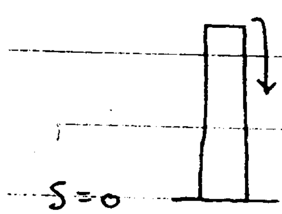
b.) strike ground : $s = 0 \rightarrow$

$0 = -16t^2 + 24t + 128 = -8(2t^2 - 3t - 16) \rightarrow$

$t = \frac{3 \pm \sqrt{9+128}}{4} = \frac{3 + \sqrt{137}}{4} \approx 3.68$ sec.

c.) $t = 3.68$ sec. $\rightarrow s' = -93.6$ ft./sec.

2.)



$s'' = -32 \rightarrow$

$s' = -32t + c \quad (t=0,\ s' = -32)$

$-32 = 0 + c \rightarrow c = -32$

$\boxed{s' = -32t - 32} \rightarrow$

$s = -16t^2 - 32t + c \quad (t=0,\ s = 128)$

$128 = 0 + c \rightarrow c = 128$

$\boxed{s = -16t^2 - 32t + 128}$;

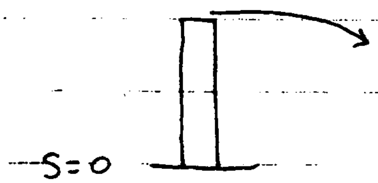
a.) strike ground : $s = 0 \rightarrow$

$0 = -16t^2 - 32t + 128 = -16(t^2 + 2t - 8) \rightarrow$

$0 = -16(t+4)(t-2) \rightarrow \quad t = 2$ sec.

b.) $t = 2$ sec. $\rightarrow \quad s' = -96$ ft./sec.

3.)



$s'' = -32 \rightarrow$

$s' = -32t + c \quad (t=0,\ s'=0 \ !!)$

$0 = 0 + c \rightarrow c = 0$

$\boxed{s' = -32t} \rightarrow$

$s = -16t^2 + c \quad (t=0,\ s=128)$

$128 = 0 + c \rightarrow c = 128$

$\boxed{s = -16t^2 + 128}$;

a.) strike ground : $s=0 \rightarrow 0 = -16t^2 + 128 \rightarrow$

$t = \sqrt{\frac{128}{16}} = \sqrt{8} \approx 2.83$ sec.

b.) $t = 2.83$ sec $\rightarrow \quad s' = -90.5$ ft./sec.

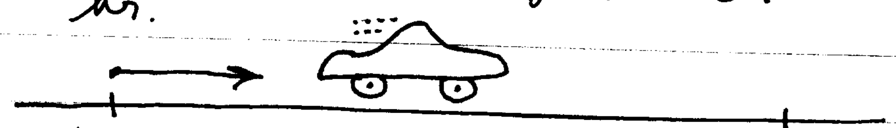
c.) As the ball drops for 2.83 seconds, its horizontal component of velocity remains constant at 100 ft./sec. Thus, the ball lands $(100)(2.83) \doteq 283$ ft. from the base of the building.

Let $s = s(t)$ be distance (ft.) traveled by car after t seconds ;

4.) $60 \frac{\text{mi.}}{\text{hr.}} = \left(\frac{60 \text{ mi.}}{\text{hr.}}\right)\left(\frac{5280 \text{ ft.}}{\text{mi.}}\right)\left(\frac{1 \text{ hr.}}{3600 \text{ sec.}}\right)$

$= 88$ ft./sec. and

$100 \frac{\text{mi.}}{\text{hr.}} \approx 146.7$ ft./sec. ;



$t = 0$ sec. $\quad\quad\quad\quad t = 15$

$s = 0$ ft. $\quad\quad\quad\quad s' = 88$ ft./sec.

$s' = 0$ ft./sec.

Assume acceleration is

$s'' = k$ ft./sec.^{2} ;

then velocity is

$s' = kt + c$; $s'(0) = 0 \rightarrow 0 = 0 + c$

$\rightarrow c = 0$, $s'(15) = 15k = 88 \Rightarrow k = 88/15$

so car's acceleration is

$$\boxed{s'' = \frac{88}{15} \approx 5.87 \text{ ft./sec.}^2} ;$$

then $\boxed{s' = \frac{88}{15} t}$ so distance

$s = \frac{44}{15} t^2 + c$; $s(0) = 0 \rightarrow 0 = 0 + c \rightarrow$

$c = 0 \longrightarrow \boxed{s = \frac{44}{15} t^2}$;

if $s' = 100$ mph ≈ 146.7 ft./sec.

then $t = \frac{88}{15} t = 146.7 \longrightarrow t = 25$ sec.

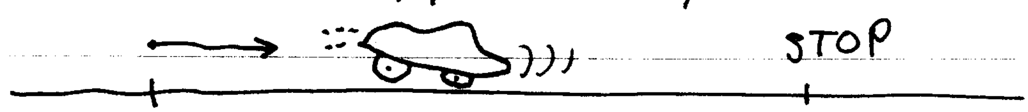
so it takes $25 - 15 = \boxed{10 \text{ sec.}}$

to go from 60 mph to 100 mph. ;

$S(25) = \frac{44}{15}(25)^2 \approx 1833$ ft.

traveled from 0 to 100 mph.

5.) Let $S = S(t)$ be distance (feet) traveled t seconds after brakes are applied ;



$t = 0$ sec.	$t = T$ sec.
$S = 0$ ft.	$S = 100$ ft.
$S' = 40$ ft./sec.	$S' = 0$ ft./sec.

assume deceleration is

$$S'' = k \text{ ft./sec.}^2 \;;$$

then velocity $S' = kt + c$; $S'(0) = 40 \rightarrow$
$40 = 0 + c \rightarrow c = 40 \rightarrow S' = kt + 40$,
$S'(T) = 0 \rightarrow \boxed{0 = kT + 40}$; ~~then~~

$$S' = kt + 40$$

so distance $S = \frac{k}{2}t^2 + 40t + c$;
$S(0) = 0 \rightarrow 0 = 0 + c \rightarrow c = 0$, $S(T) = 100 \rightarrow$
$\boxed{100 = \frac{k}{2}T^2 + 40T}$; ~~solve the~~

system of equations : $kT = -40$
~~so~~ $100 = \frac{1}{2}(kT)T + 40T$
$= \frac{1}{2}(-40)T + 40T = 20T \rightarrow$

$T = 5 \text{ sec.}$ to stop the car;

and $KT = -40 \rightarrow 5K = -40 \rightarrow$

car's deceleration is

$s'' = k = -8 \text{ ft./sec.}^2$