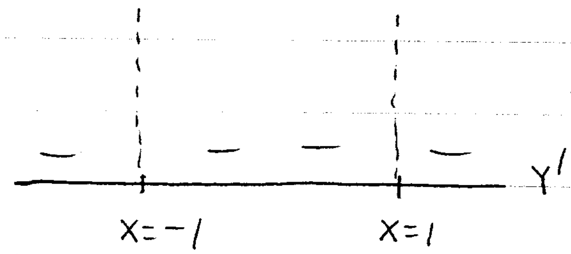
HW #14

Section 4.2

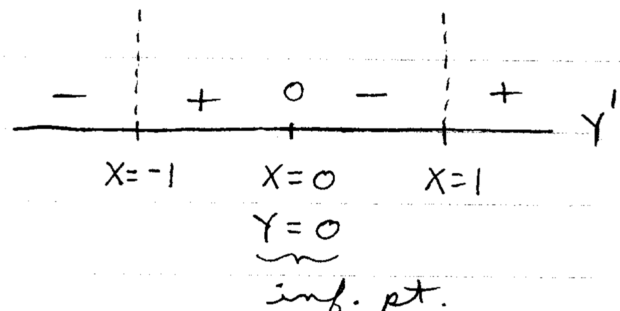
$\boxed{184:28}$ $Y = \dfrac{x}{x^2-1}, \quad x \neq 1, -1$

$Y' = \dfrac{(x^2-1)(1) - x(2x)}{(x^2-1)^2} = \dfrac{-1-x^2}{(x^2-1)^2} = 0,$



$$Y'' = \frac{(x^2-1)^2(-2x) - (-1-x^2)\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4}$$

$$= \frac{-2x(x^2-1)\cdot[(x^2-1) - (1+x^2)\cdot 2]}{(x^2-1)^4} = \frac{2x(3+x^2)}{(x^2-1)^3} = 0,$$



Y is $\downarrow$ for $x<-1$, $-1<x<1$, $x>1$,

Y is $\cup$ for $-1<x<0$, $x>1$,

Y is $\cap$ for $x<-1$, $0<x<1$;

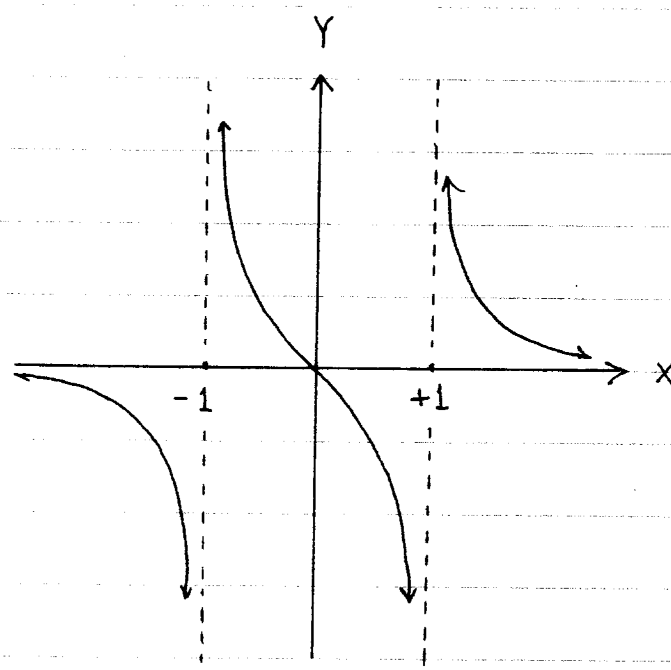
$\displaystyle\lim_{x\to\pm\infty} \frac{x}{x^2-1} = \lim_{x\to\pm\infty} \frac{1}{x-\frac{1}{x}} = 0,$

$\displaystyle\lim_{x\to 1^+} \frac{x}{x^2-1} = \text{"}\frac{1}{0^+}\text{"} = +\infty,$

$\displaystyle\lim_{x\to 1^-} \frac{x}{x^2-1} = \text{"}\frac{1}{0^-}\text{"} = -\infty,$

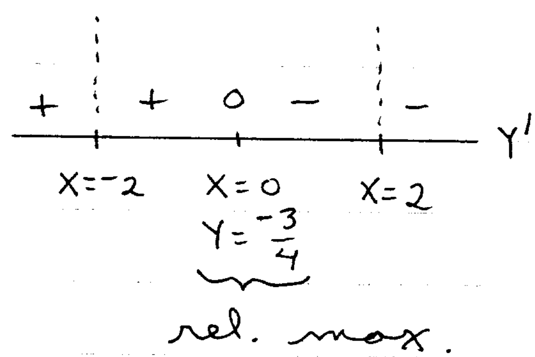
$\displaystyle\lim_{x\to -1^+} \frac{x}{x^2-1} = \text{"}\frac{-1}{0^-}\text{"} = +\infty,$

$\displaystyle\lim_{x\to -1^-} \frac{x}{x^2-1} = \text{"}\frac{-1}{0^+}\text{"} = -\infty;$



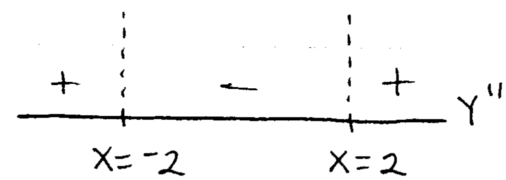
$\boxed{184:31}$ $Y = \dfrac{x^2+3}{x^2-4}$, $x \neq 2, -2$

$$Y' = \frac{(x^2-4)(2x)-(x^2+3)(2x)}{(x^2-4)^2} = \frac{-14x}{(x^2-4)^2} = 0,$$



$$Y'' = \frac{(x^2-4)^2(-14)-(-14x)\cdot 2(x^2-4)(2x)}{(x^2-4)^4}$$

$$= \frac{-14(x^2-4)\left[(x^2-4)-4x^2\right]}{(x^2-4)^4} = \frac{14(4+3x^2)}{(x^2-4)^3} = 0,$$



Y is ↑ for $x<-2$, $-2<x<0$,
Y is ↓ for $0<x<2$, $x>2$,
Y is ∪ for $x<-2$, $x>2$,
Y is ∩ for $-2<x<2$;

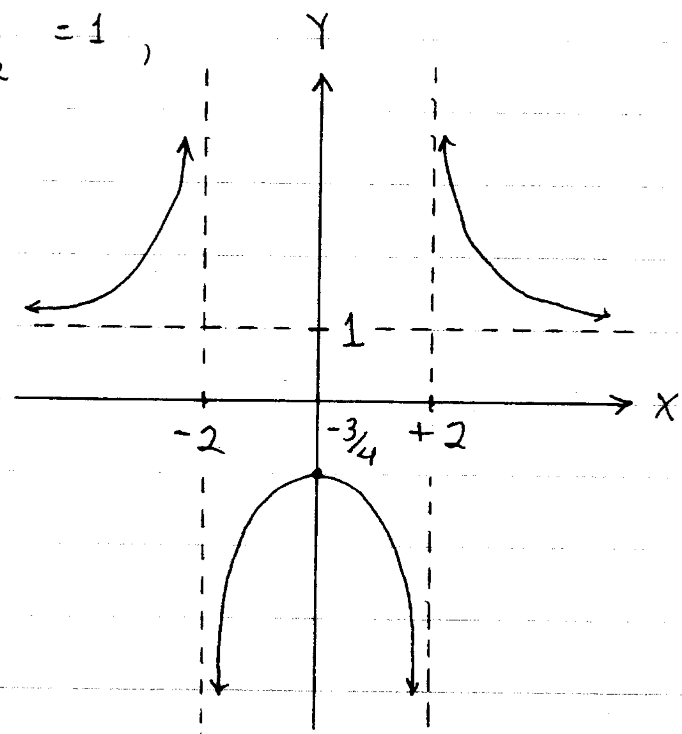
$$\lim_{x\to\pm\infty} \frac{x^2+3}{x^2-4} = \lim_{x\to\pm\infty} \frac{1+3/x^2}{1-4/x^2} = 1,$$

$$\lim_{x\to 2^+} \frac{x^2+3}{x^2-4} = \text{"}\frac{7}{0^+}\text{"} = +\infty,$$

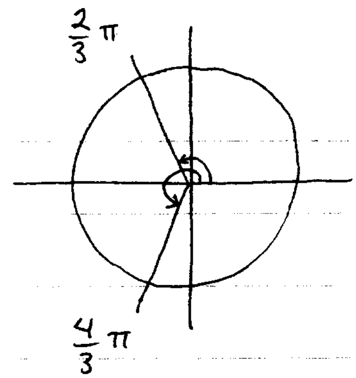
$$\lim_{x\to 2^-} \frac{x^2+3}{x^2-4} = \text{"}\frac{7}{0^-}\text{"} = -\infty,$$

$$\lim_{x\to -2^+} \frac{x^2+3}{x^2-4} = \text{"}\frac{7}{0^-}\text{"} = -\infty,$$

$$\lim_{x\to -2^-} \frac{x^2+3}{x^2-4} = \text{"}\frac{7}{0^+}\text{"} = +\infty.$$



$\boxed{184:43}$ $Y = \dfrac{\sin x}{1 + 2\cos x}$, $\cos x \neq \frac{-1}{2}$,



$x \neq \pm\frac{2}{3}\pi, \pm\frac{4}{3}\pi, \pm\frac{8}{3}\pi, \pm\frac{10}{3}\pi, \cdots,$

$$Y' = \frac{(1+2\cos x)(\cos x) - \sin x\,(-2\sin x)}{(1+2\cos x)^2}$$

$$= \frac{\cos x + 2\cos^2 x + 2\sin^2 x}{(1+2\cos x)^2}$$

$$= \frac{2+\cos x}{(1+2\cos x)^2} = 0,$$

Y': $+ \mid + \mid + \mid + \mid +$ at $x = -\frac{4}{3}\pi$, $x = -\frac{2}{3}\pi$, $x = \frac{2}{3}\pi$, $x = \frac{4}{3}\pi$

$$Y'' = \frac{(1+2\cos x)^2 \cdot (-\sin x) - (2+\cos x)\cdot 2(1+2\cos x)\cdot(-2\sin x)}{(1+2\cos x)^4}$$

$$= \frac{-\sin x \cdot (1+2\cos x)\cdot\left[(1+2\cos x) - 4(2+\cos x)\right]}{(1+2\cos x)^4}$$

$$= \frac{\sin x\cdot(7+2\cos x)}{(1+2\cos x)^3} = 0,$$

Y'': $+ \mid -\ 0\ + \mid -\ 0\ + \mid -\ 0\ + \mid -$ at $x = -\frac{4}{3}\pi$, $x = -\frac{2}{3}\pi$, $x = \frac{2}{3}\pi$, $x = \frac{4}{3}\pi$

$\sin x = 0$; inf. pts. $\left\{ \begin{array}{lll} x = -\pi & x = 0 & x = \pi \\ Y = 0 & Y = 0 & Y = 0 \end{array} \right.$

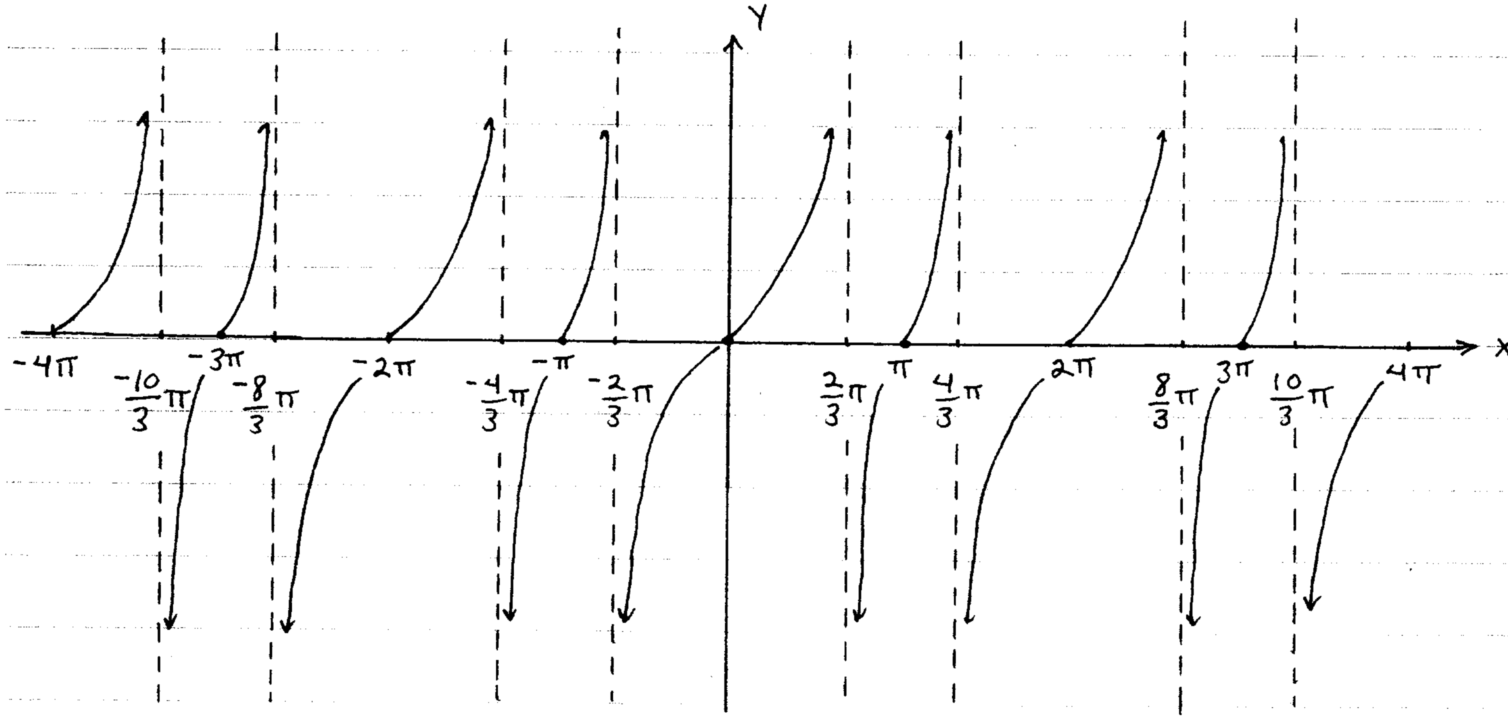
$$\lim_{x \to \frac{2}{3}\pi^+} \frac{\sin x}{1+2\cos x} = \text{"}\frac{\frac{\sqrt{3}}{2}}{0^-}\text{"} = -\infty,$$

$$\lim_{x \to \frac{2}{3}\pi^-} \frac{\sin x}{1+2\cos x} = \text{"}\frac{\frac{\sqrt{3}}{2}}{0^+}\text{"} = +\infty,$$

$$\lim_{x \to -\frac{2}{3}\pi^+} \frac{\sin x}{1+2\cos x} = \text{"}\frac{-\frac{\sqrt{3}}{2}}{0^+}\text{"} = -\infty,$$

$$\lim_{x \to -\frac{2}{3}\pi^-} \frac{\sin x}{1 + 2\cos x} = \text{"}\frac{-\frac{\sqrt{3}}{2}}{0^-}\text{"} = +\infty, \quad etc.;$$

$x = 0 : y = 0$ and $y = 0 : \sin x = 0$



$\boxed{184:44}$ $\quad y = \frac{\sqrt{x^2-1}}{x}, \quad x \geq 1, x \leq -1,$

$$y' = \frac{x \cdot \frac{1}{2}(x^2-1)^{-1/2} \cdot (2x) - (x^2-1)^{1/2}(1)}{x^2} = \frac{\frac{x^2}{(x^2-1)^{1/2}} - \frac{(x^2-1)^{1/2}}{1}}{\frac{x^2}{1}}$$

$$= \frac{1}{x^2(x^2-1)^{1/2}} = 0,$$

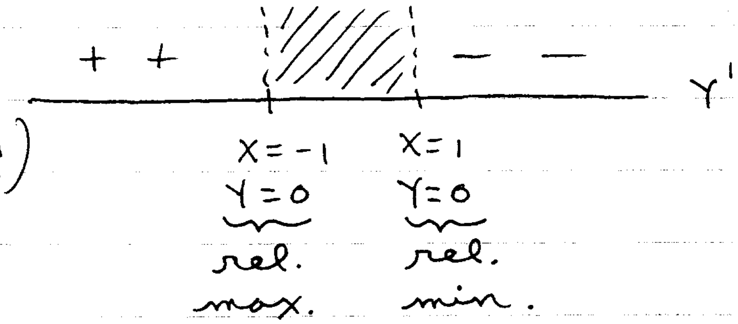
$+ \quad + \quad | \quad | \quad + \quad +$ $\quad y'$

rel. max. $\left\{ \begin{array}{l} x = -1 \\ y = 0 \end{array} \right.$ $\quad$ $\left. \begin{array}{l} x = 1 \\ y = 0 \end{array} \right\}$ rel. min.

$$y'' = \frac{-\left[x^2 \cdot \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x + 2x(x^2-1)^{1/2}\right]}{x^4 \; (x^2-1)} = \cdots = \frac{2-3x^2}{x^3(x^2-1)^{3/2}} = 0,$$

$2-3x^2=0 \rightarrow$

$x=\pm\sqrt{2/3}$ (NOT in domain!)



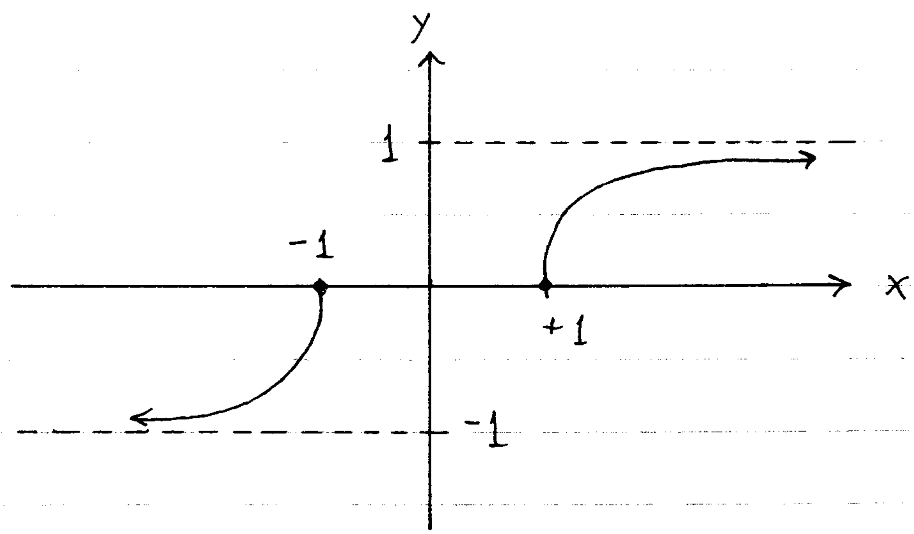
Y is $\uparrow$ for $x<-1$, $x>1$,

Y is $\cup$ for $x<-1$,

Y is $\cap$ for $x>1$;

$$\lim_{x\to+\infty}\frac{\sqrt{x^2-1}}{x}=\lim_{x\to+\infty}\sqrt{\frac{x^2-1}{x^2}}=\lim_{x\to+\infty}\sqrt{1-\frac{1}{x^2}}=+1,$$

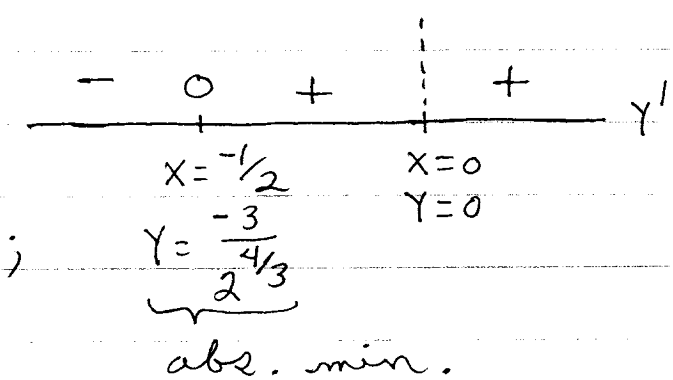
$$\lim_{x\to-\infty}\frac{\sqrt{x^2-1}}{x}=\lim_{x\to-\infty}-\sqrt{\frac{x^2-1}{x^2}}=\lim_{x\to-\infty}-\sqrt{1-\frac{1}{x^2}}=-1;$$



$\boxed{184:47}$ $Y=2x^{1/3}+x^{4/3}$,

$$Y'=\frac{2}{3}x^{-2/3}+\frac{4}{3}x^{1/3}$$

$$=\frac{2}{3x^{2/3}}+\frac{4x^{1/3}}{3}=\frac{2+4x}{3x^{2/3}}=0;$$



$$y'' = \frac{3x^{2/3}\cdot(4) - (2+4x)\cdot 2x^{-1/3}}{9x^{4/3}} = \frac{\frac{12x^{2/3}}{1} - \frac{4+8x}{x^{1/3}}}{\frac{9x^{4/3}}{1}}$$

$$= \cdots = \frac{4x-4}{9x^{5/3}} = 0;$$

$+$	$\vdots$	$-$	0	$+$	y''
	$x=0$		$x=1$		
	$y=0$		$y=3$		

inf. pts.

y is $\uparrow$ for $-\frac{1}{2} < x < 0$, $x > 0$,

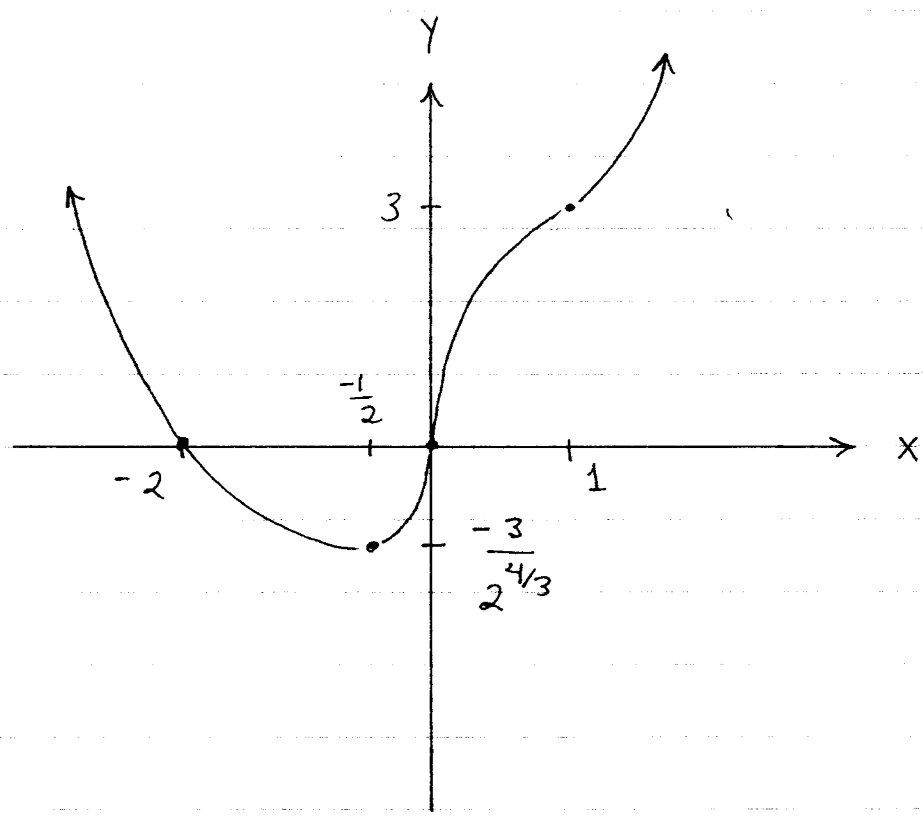
y is $\downarrow$ for $x < -1/2$,

y is $\cup$ for $x < 0$, $x > 1$,

y is $\cap$ for $0 < x < 1$;

$x=0$: $y=0$

$y=0$: $0 = 2x^{1/3} + x^{4/3} = x^{1/3}(2+x) \rightarrow x=0, x=-2$;



Section 4.5

$\boxed{203:7}$ $Y = X^4 - 4X^3$

$Y' = 4X^3 - 12X^2 = 4X^2(X-3) = 0;$

$Y'' = 12X^2 - 24X = 12X(X-2) = 0;$

$-$	0	$-$	0	$+$	Y'
	$X=0$		$X=3$		
	$Y=0$		$\underbrace{Y=-27}_{\text{abs. min.}}$		

	$+$	0	$-$	0	$+$	Y''
inf. pts.		$X=0$, $Y=0$		$X=2$, $Y=-16$		

Y is $\uparrow$ for $X > 3$,

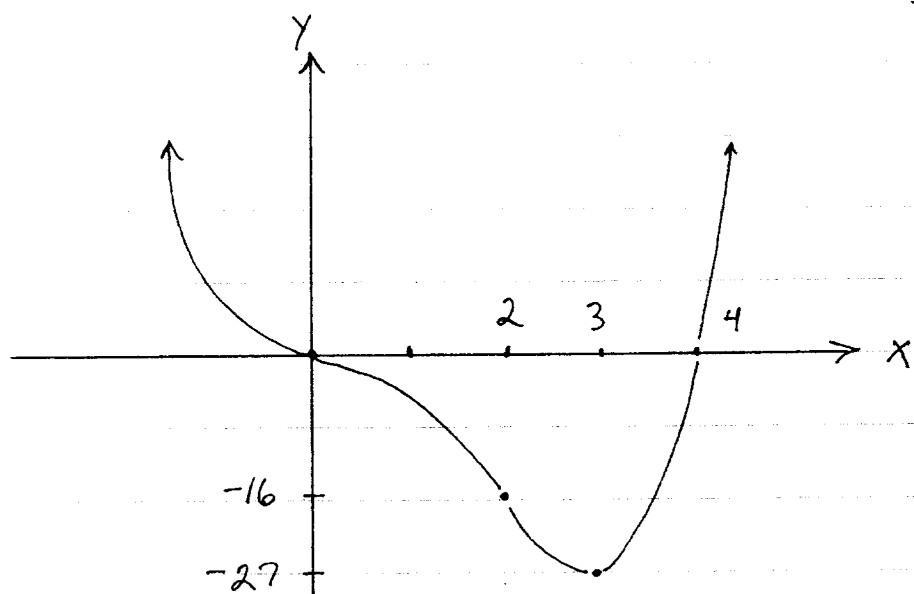
Y is $\downarrow$ for $X < 3$,

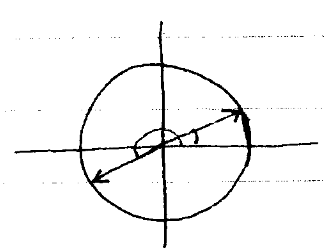
Y is $\cup$ for $X<0$, $X>2$,

Y is $\cap$ for $0<X<2$;

$X=0 : Y=0$ and

$Y=0 : 0 = X^4 - 4X^3 = X^3(X-4) = 0 \rightarrow X=0, X=4;$





$\boxed{203:14}$ $Y = \sin X + \sqrt{3}\cos X$

$Y' = \cos X - \sqrt{3}\sin X = 0 \rightarrow$

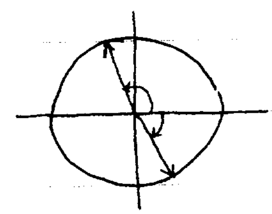
$\dfrac{\sin X}{\cos X} = \dfrac{1}{\sqrt{3}} = \dfrac{\frac{1}{2}}{\sqrt{3}/2} = \dfrac{-\frac{1}{2}}{-\sqrt{3}/2} \rightarrow X = \dfrac{\pi}{6} \pm n\pi$, n an integer;

$-\quad 0 \quad + \quad 0 \quad - \quad 0 \quad + \quad 0 \quad - \quad 0 \quad + \quad 0 \quad -$ $\quad Y'$

$X = \frac{-17\pi}{6}$	$X = \frac{-11\pi}{6}$	$X = \frac{-5\pi}{6}$	$X = \frac{\pi}{6}$	$X = \frac{7\pi}{6}$	$X = \frac{13\pi}{6}$
$Y = -2$	$Y = 2$	$Y = -2$	$Y = 2$	$Y = -2$	$Y = 2$
abs. min.	abs. max.	abs. min.	abs. max	abs. min.	abs. max.

$$Y'' = -\sin X - \sqrt{3}\cos X = 0 \rightarrow \frac{\sin X}{\cos X} = -\sqrt{3} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} \rightarrow$$

$X = \frac{2}{3}\pi \pm n\pi$, n an integer;

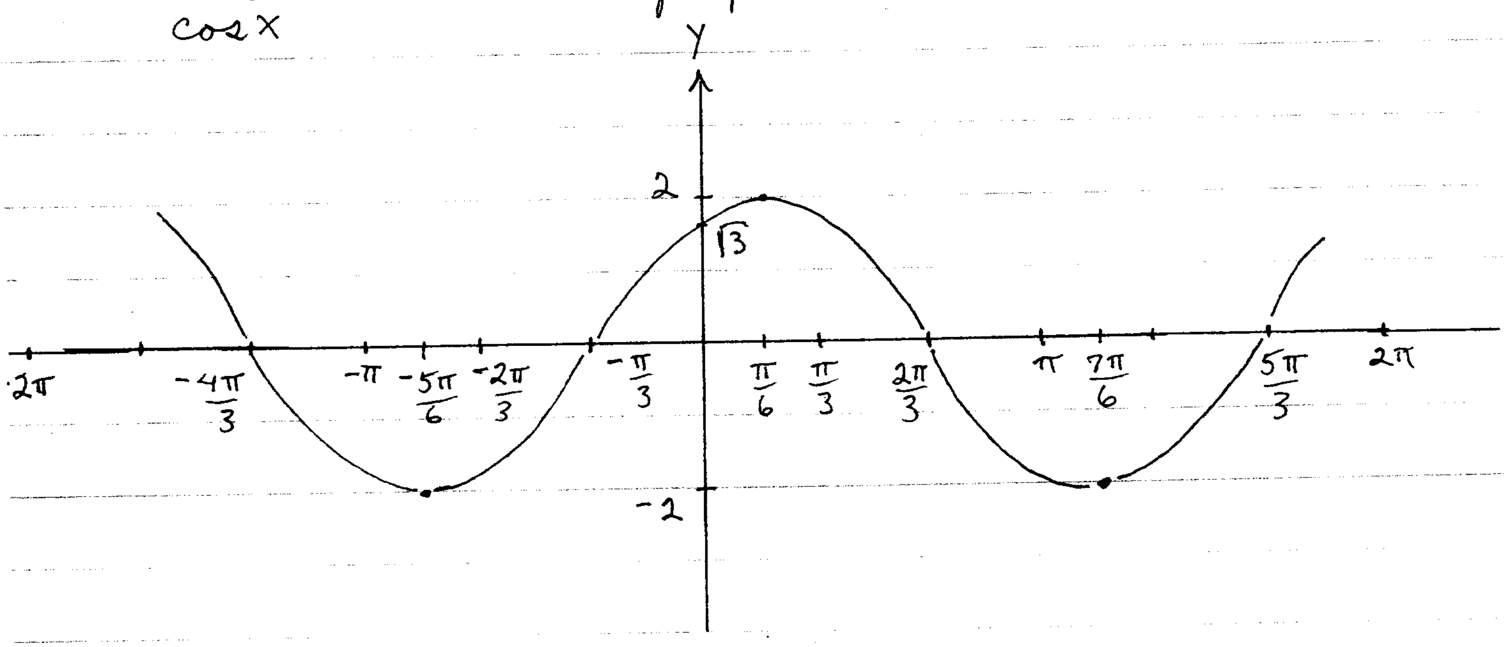


$-\quad 0 \quad + \quad 0 \quad - \quad 0 \quad + \quad 0 \quad - \quad 0 \quad +$

$X = \frac{-4\pi}{3}$	$X = \frac{-\pi}{3}$	$X = \frac{2\pi}{3}$	$X = \frac{5\pi}{3}$	$X = \frac{8\pi}{3}$	all are inf. pts.
$Y = 0$	$Y = 0$	$Y = 0$	$Y = 0$	$Y = 0$	

$X = 0$: $Y = \sqrt{3}$ and $Y = 0$: $0 = \sin X + \sqrt{3}\cos X \rightarrow$

$\frac{\sin X}{\cos X} = -\sqrt{3}$ so inf. pts. are intercepts;



$\boxed{203:11}$ $Y = x^3 - 6x^2 - 15x \longrightarrow$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ \hline & x=-1 & & x=5 & \\ & \underbrace{Y=8}_{\text{rel. max.}} & & \underbrace{Y=-100}_{\text{rel. min.}} & \end{array} \quad Y'$$

$Y' = 3x^2 - 12x - 15$

$= 3(x^2 - 4x - 5) = 3(x-5)(x+1) = 0$

$Y'' = 6x - 12 = 6(x-2) = 0$

$$\begin{array}{ccc} - & 0 & + \\ \hline & x=2 & \\ & \underbrace{Y=-46}_{\text{infl. pt.}} & \end{array} \quad Y''$$

Y is $\uparrow$ for $x<-1, x>5$

Y is $\downarrow$ for $-1<x<5$

Y is $\cup$ for $x>2$

Y is $\cap$ for $x<2$

$x=0$: $Y=0$

$Y=0$: $x^3 - 6x^2 - 15x = x(x^2 - 6x - 15) = 0 \longrightarrow x=0$ or

$$x = \frac{6 \pm \sqrt{36+60}}{2} = \frac{6 \pm \sqrt{96}}{2} = 3 \pm 2\sqrt{6}$$