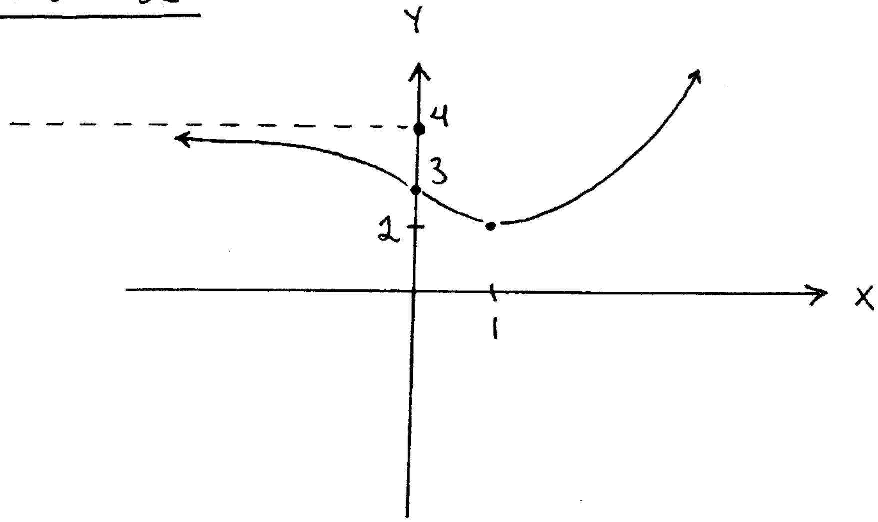
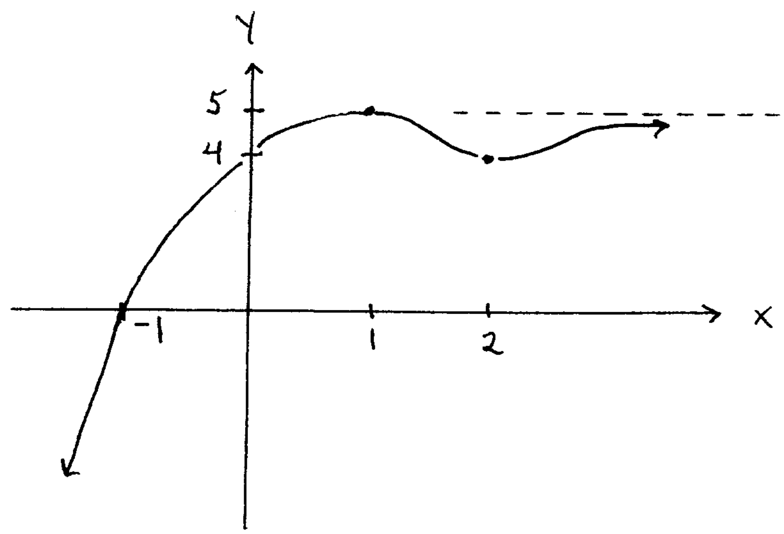
HW #15

Section 4.2

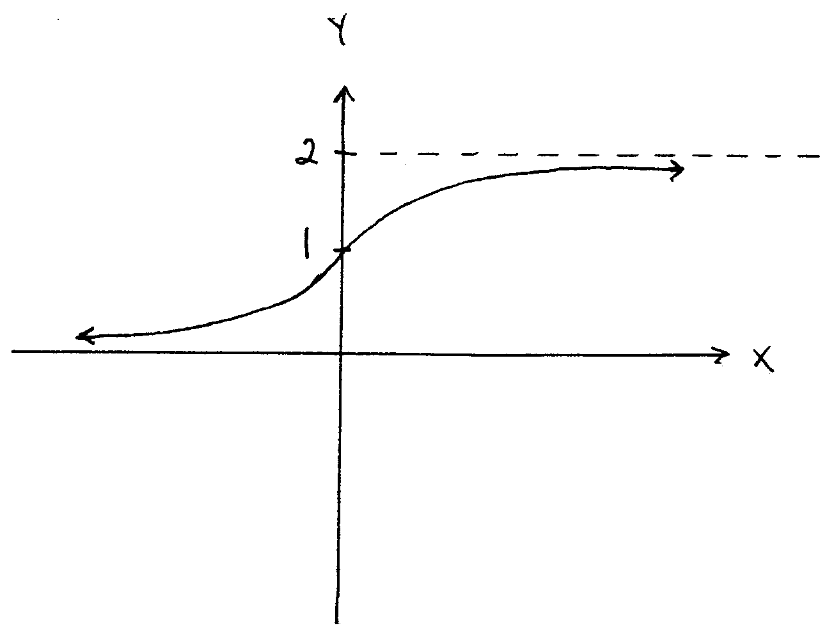
$\boxed{184:12}$



$\boxed{184:13}$



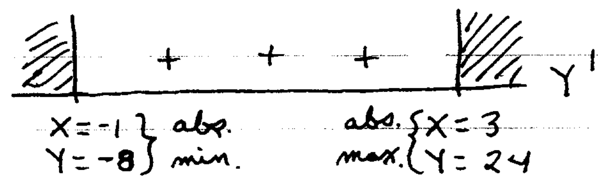
$\boxed{184:16}$



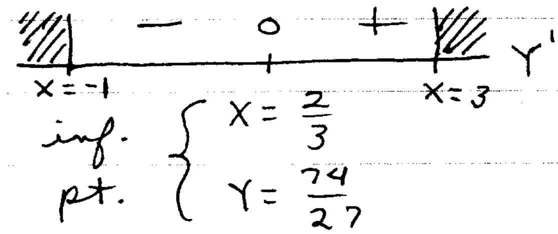
$\boxed{184:37}$ $Y = x^3 - 2x^2 + 5x$ on $[-1, 3]$,

$Y' = 3x^2 - 4x + 5 = 0 \rightarrow$

$x = \dfrac{4 \pm \sqrt{16-60}}{6}$ (complex!);



$Y'' = 6x - 4 = 0$;



$X=0: \ Y=0$

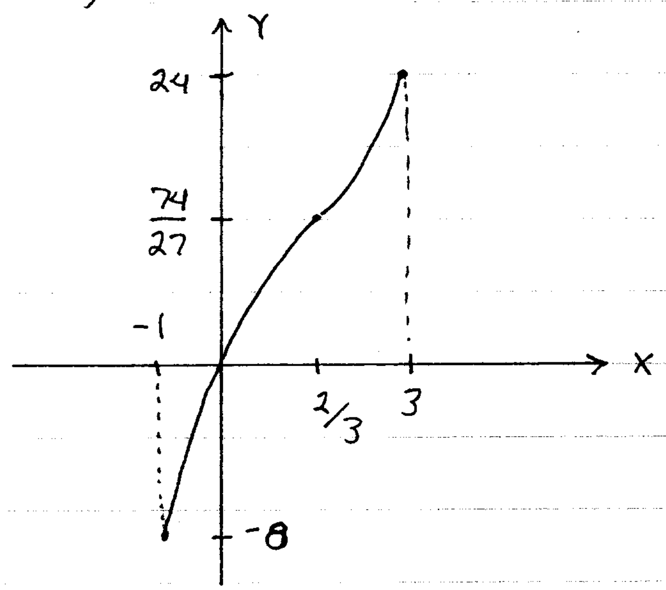
$Y=0: \ 0 = x^3 - 2x^2 + 5x$

$= x(x^2 - 2x + 5) \longrightarrow x = 0, \ x = \dfrac{2 \pm \sqrt{4-20}}{2}$ (complex!),

Y is $\uparrow$ for all x-values,

Y is $\cup$ for $x > 2/3$,

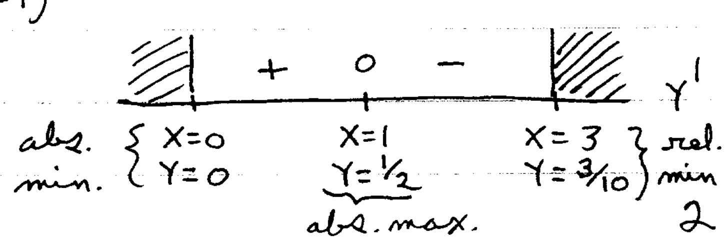
Y is $\cap$ for $x < 2/3$.



$\boxed{184:38}$ $Y = \dfrac{x}{x^2+1}$ on $[0, 3]$,

$Y' = \dfrac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$

$= \dfrac{(1-x)(1+x)}{(x^2+1)^2} = 0$;

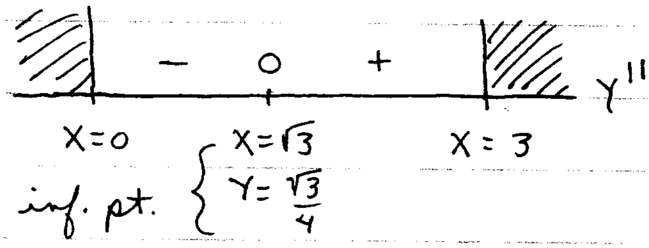


$$y'' = \frac{(x^2+1)^2(-2x) - (1-x^2)\cdot 2(x^2+1)(2x)}{(x^2+1)^4}$$

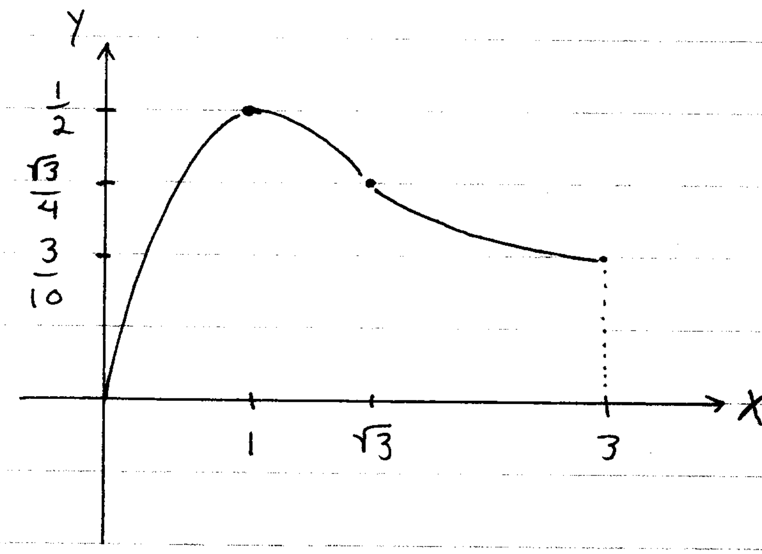
$$= \frac{-2x(x^2+1)\left[(x^2+1) + 2(1-x^2)\right]}{(x^2+1)^4}$$

$$= \frac{-2x(3-x^2)}{(x^2+1)^3}$$

$$= \frac{-2x(\sqrt{3}-x)(\sqrt{3}+x)}{(x^2+1)^3} = 0$$



y is $\uparrow$ for $0 < x < 1$,
y is $\downarrow$ for $1 < x < 3$,
y is $\cup$ for $\sqrt{3} < x < 3$,
y is $\cap$ for $0 < x < \sqrt{3}$,
$x = 0$: $y = 0$
$y = 0$: $x = 0$

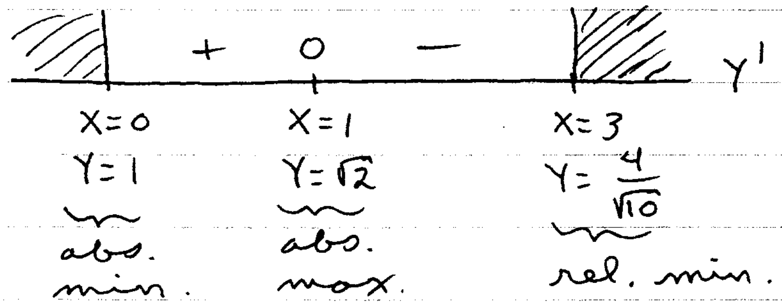


$\boxed{184:40}$ $y = \dfrac{x+1}{(x^2+1)^{1/2}}$ on $[0,3]$,

$$y' = \frac{(x^2+1)^{1/2}(1) - (x+1)\cdot \frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}}(\cancel{2}x)}{x^2+1}$$

$$= \frac{\dfrac{(x^2+1)^{1/2}}{1} - \dfrac{x(x+1)}{(x^2+1)^{1/2}}}{\dfrac{x^2+1}{1}} = \frac{\cancel{x^2}+1-\cancel{x^2}-x}{(x^2+1)^{1/2}} \cdot \frac{1}{x^2+1}$$

$$= \frac{1-x}{(x^2+1)^{3/2}} = 0 \;;$$

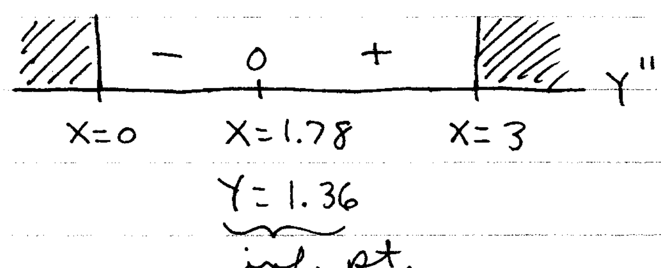


$$Y'' = \frac{(x^2+1)^{3/2}(-1) - (1-x)\cdot\frac{3}{\cancel{2}}(x^2+1)^{1/2}\cdot \cancel{2}x}{(x^2+1)^3}$$

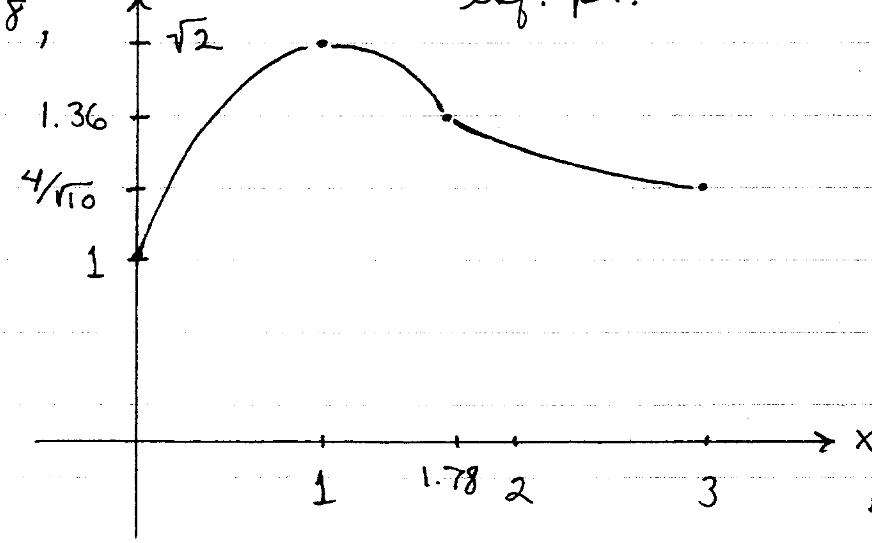
$$= \frac{-(x^2+1)^{1/2}\left[(x^2+1)+3x(1-x)\right]}{(x^2+1)^3}$$

$$= \frac{-(-2x^2+3x+1)}{(x^2+1)^{5/2}} = \frac{2x^2-3x-1}{(x^2+1)^{5/2}} = 0 \rightarrow$$

$$x = \frac{3 \pm \sqrt{9+8}}{4} = \frac{3\pm\sqrt{17}}{4} = 1.78 \text{ or } \cancel{-.28}$$



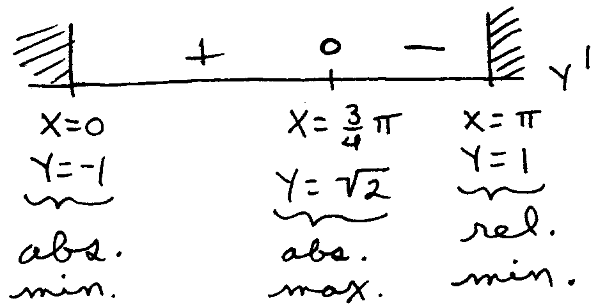
Y is $\uparrow$ for $0<x<1$,
Y is $\downarrow$ for $1<x<3$,
Y is $\cup$ for $1.78<x<3$,
Y is $\cap$ for $0<x<1.78$,
$x=0$: $Y=1$
$Y=0$: $x=-1$ (No!)



$\boxed{184:42}$ $Y = \sin X - \cos X$ on $[0, \pi]$

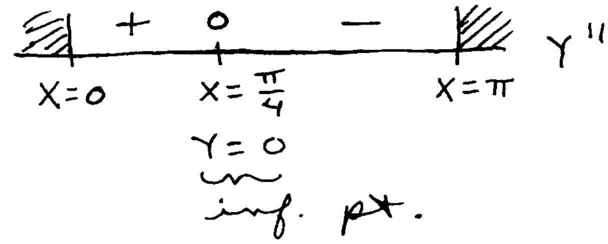
$\rightarrow Y' = \cos X + \sin X = 0$

$\rightarrow \cos X = -\sin X$



$\rightarrow Y'' = -\sin X + \cos X = 0$

$\rightarrow \cos X = \sin X$



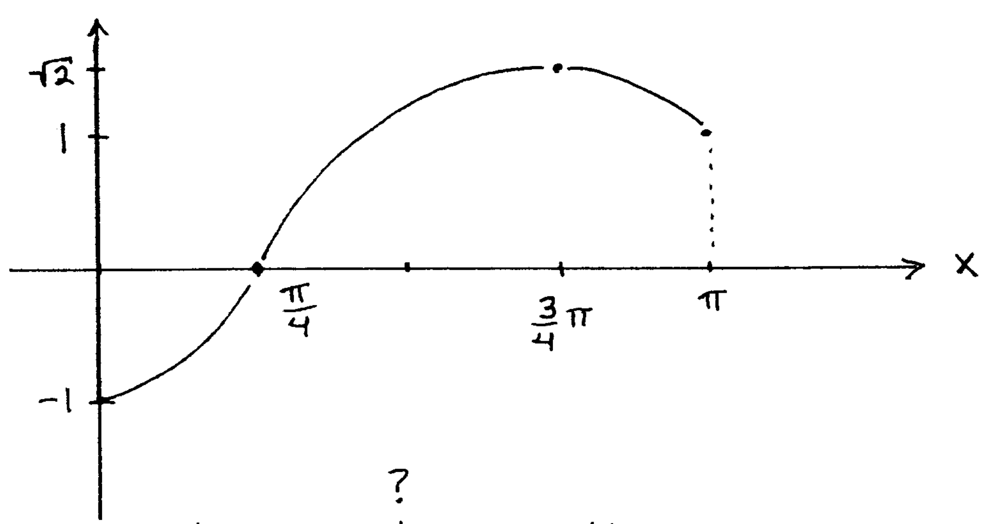
Y is $\uparrow$ for $0 < X < \frac{3}{4}\pi$,

Y is $\downarrow$ for $\frac{3}{4}\pi < X < \pi$,

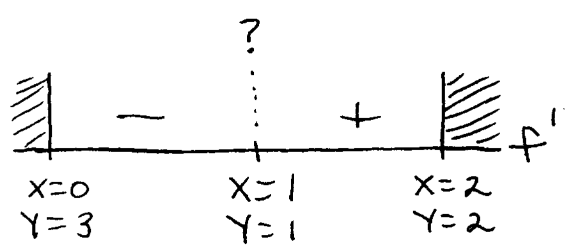
Y is $\cup$ for $0 < X < \frac{\pi}{4}$,

Y is $\cap$ for $\frac{\pi}{4} < X < \pi$,

$X = 0 : Y = -1$ and $Y = 0 : X = \frac{\pi}{4}$



$\boxed{184:52}$



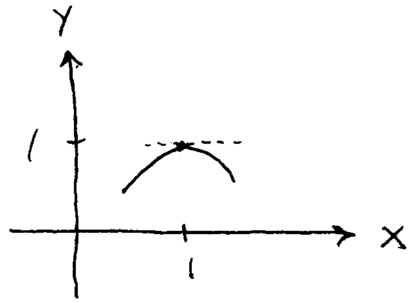
f differentiable so f continuous,

a.) $Y = 1$ is min. value on $[0, 2]$.

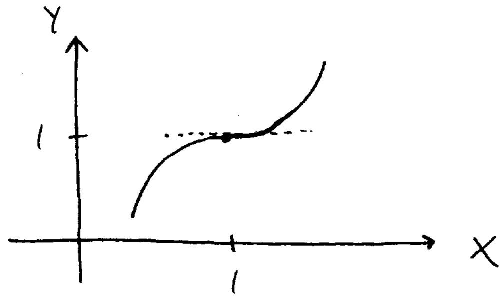
b.) $Y = 3$ is max. value on $[0, 2]$.

Section 4.5

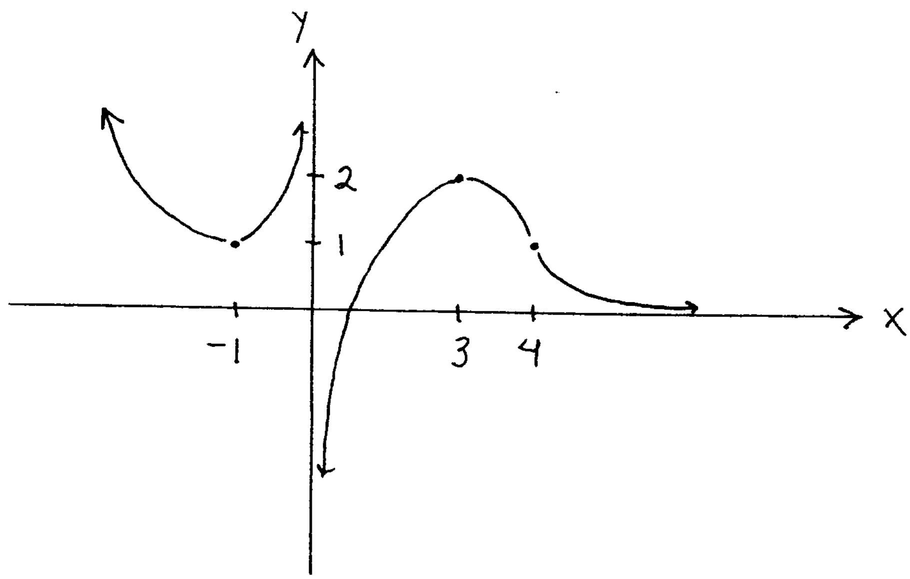
$\boxed{203:22}$



$\boxed{203:24}$



$\boxed{203:32}$



$\boxed{203:34}$

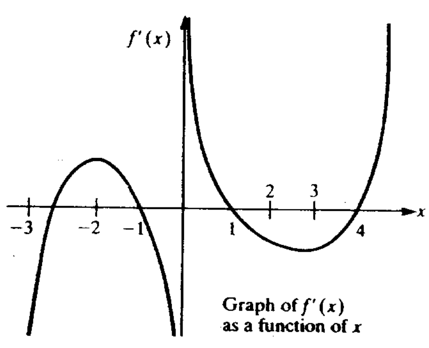


Figure 14

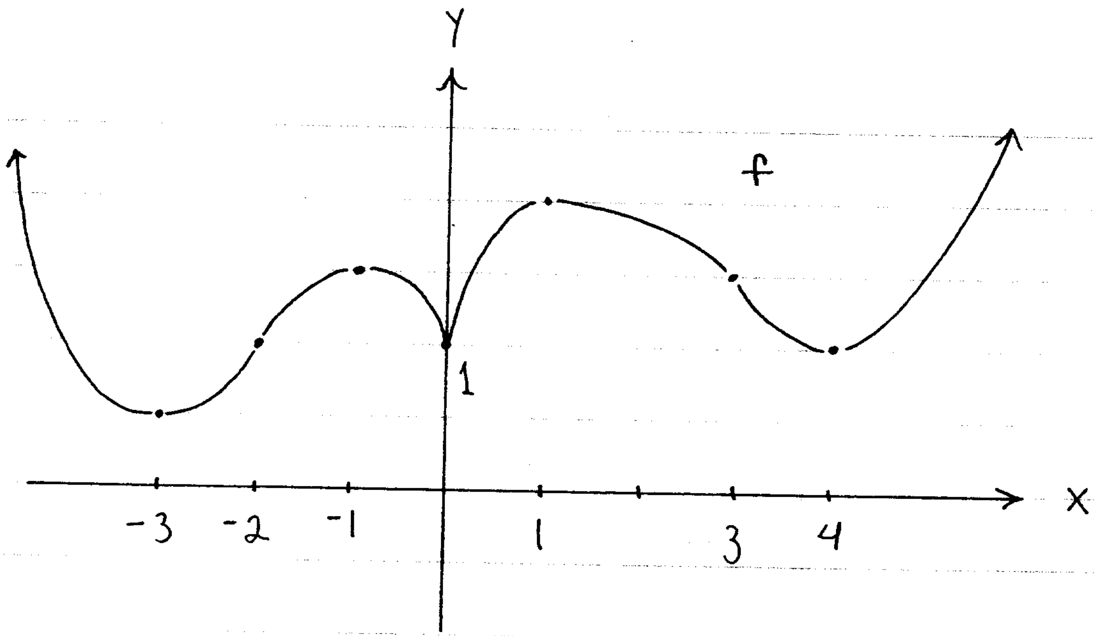
a.) f is ↑ where f' is + :
$-3<x<-1$, $0<x<1$, $x>4$;
f is ↓ where f' is − :
$x<-3$, $-1<x<0$, $1<x<4$;
f is ∪ where $f''=(f')'$ is + :
$x<-2$, $x>3$
f is ∩ where $f''=(f')'$ is − :
$-2<x<0$, $0<x<3$.

critical #'s ($f' = 0$) : $x = -3$, $x = -1$, $x = 1$, $x = 4$;

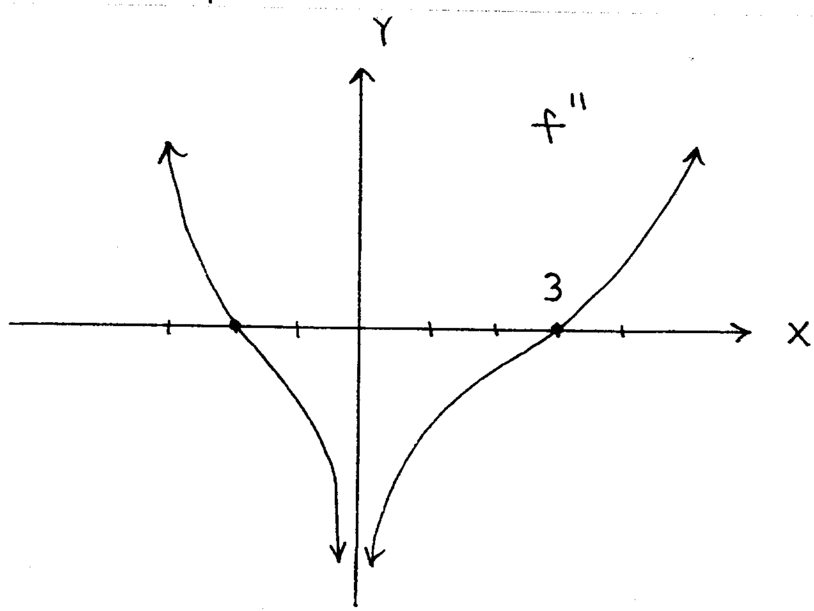
$-$	0	$+$	0	$-$		$+$	0	$-$	0	$+$	f'
	$x=-3$		$x=-1$		$x=0$		$x=1$		$x=4$		
	min.		max.		min.		max.		min.		

$+$	0	$-$		$-$	0	$+$	f''
	$x=-2$		$x=0$		$x=3$		
	inf. pt.				inf. pt.		

b.)



c.)



$\boxed{203:38}$ $\quad \dfrac{dY}{dt} = kY(M-Y) \rightarrow$

$$\frac{d^2Y}{dt^2} = (kY)\left(-\frac{dY}{dt}\right) + k\frac{dY}{dt}\cdot(M-Y)$$

$$= k\cdot\frac{dY}{dt}\cdot\left[M-2Y\right] = 0 \rightarrow$$

$k=0$ (NO!) or $\dfrac{dY}{dt}=0$ (NO because Y is growing!) or $M-2Y=0 \rightarrow Y=\dfrac{M}{2}$:

$+ \quad 0 \quad -$ $\quad Y''$

determines inflection pt. $\longrightarrow Y = \dfrac{M}{2}$

$\boxed{203:39}$ $\quad f''(x) = (x-1)(x-2)$

$+ \quad 0 \quad - \quad 0 \quad +$ $\quad f''$

$x=1 \qquad x=2$

a.) f is $\cup$ for $x<1$, $x>2$

b.) f is $\cap$ for $1<x<2$

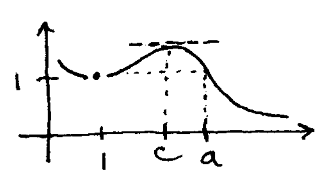
c.) inflection numbers: $x=1$, $x=2$

d.) $f''(x) = x^2-3x+2 \rightarrow$

$f'(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$ "works" $\rightarrow$

$f(x) = \frac{1}{12}x^4 - \frac{1}{2}x^3 + x^2$ "works"

$\boxed{203:41}$ $\quad (1,1)$ only critical point, $\lim\limits_{x\to\infty} f(x) = 0$, f'' and f' are continuous :



a.) f must be $\downarrow$ for $x>1$: if $f\uparrow$ then $\downarrow$ there is some # a satisfying $f(a)=1$, then by Rolle's Theorem a # c satisfies $f'(c)=0$, a contradiction since $x=1$ is the only critical number.

b.) f must have an inflection point: since $(1,1)$ is critical point, then $f'(1) = 0$ and from part a.) f is $\downarrow$ for $x > 1$. Necessarily, f must be concave down at x-values near $x = 1$ and larger than $x = 1$. If the concavity does not change, then the graph of f would not satisfy $\lim_{x \to \infty} f(x) = 0$. Where the concavity changes is the inflection point.