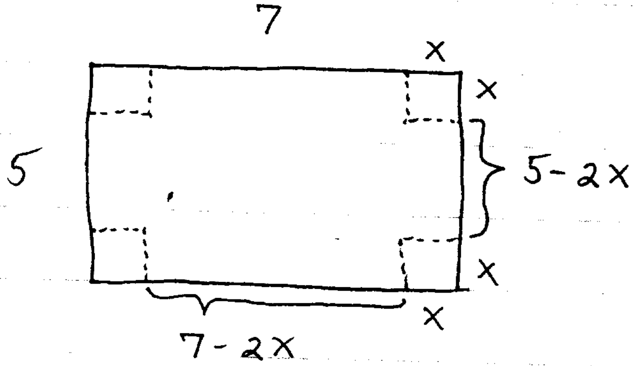
HW #16

Section 4.7

$\boxed{216:4}$



maximize volume

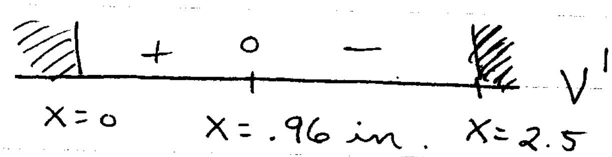
$V = x(5-2x)(7-2x) \rightarrow$

$V' = (1)(5-2x)(7-2x) + x(-2)(7-2x) + x(5-2x)(-2)$

$= 35 - 24x + 4x^2 - 14x + 4x^2 + -10x + 4x^2$

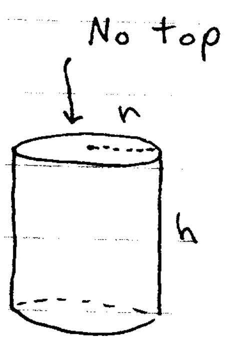
$= 12x^2 - 48x + 35 = 0 \rightarrow$

$x = \dfrac{48 \pm \sqrt{48^2 - 48(35)}}{24} = \cancel{3.04}$ or $.96 \rightarrow$



$V = 15.02\ in^3$

$\boxed{216:8}$



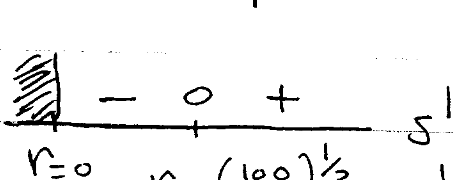
$\pi r^2 h = 100 \rightarrow h = \dfrac{100}{\pi r^2}$,

minimize surface area

$S = \pi r^2 + 2\pi r h = \pi r^2 + 2\pi r\left(\dfrac{100}{\pi r^2}\right) = \pi r^2 + \dfrac{200}{r} \rightarrow$

$S' = 2\pi r - \dfrac{200}{r^2} = \dfrac{2\pi r^3 - 200}{r^2} = 0 \rightarrow 2\pi r^3 - 200 = 0 \rightarrow$

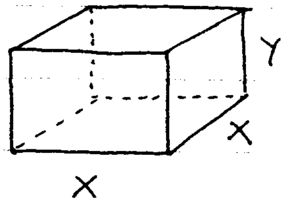
$r = \left(\dfrac{100}{\pi}\right)^{\frac{1}{3}}$



$r = \left(\frac{100}{\pi}\right)^{1/3}$, $h = \left(\frac{100}{\pi}\right)^{1/3}$, $S = 3\left(100^{2/3}\right)\pi^{1/3}$

≈ 3.17 in. ≈ 3.17 in. $\approx 94.7\ in^2$

$\boxed{216:9}$



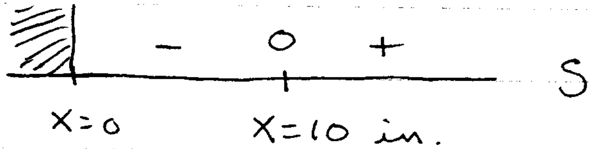
$x^2 y = 1000 \rightarrow y = \frac{1000}{x^2}$,

minimize surface area

$S = 2x^2 + 4xy = 2x^2 + 4x\left(\frac{1000}{x^2}\right) = 2x^2 + \frac{4000}{x} \rightarrow$

$S' = 4x - \frac{4000}{x^2} = \frac{4x^3 - 4000}{x^2} = \frac{4(x^3 - 1000)}{x^2} = 0 \rightarrow$

$x = 10$;

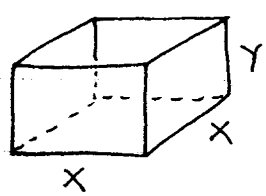


$x = 10$ in.

$y = 10$ in.

$S = 600$ in.^{2}

$\boxed{216:10}$



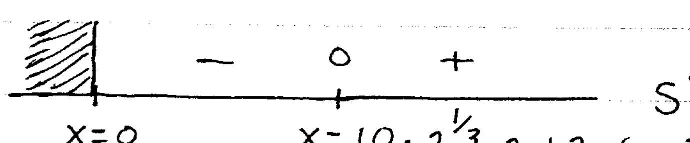
$x^2 y = 1000 \rightarrow y = \frac{1000}{x^2}$,

minimize surface area

$S = x^2 + 4xy = x^2 + 4x\left(\frac{1000}{x^2}\right) = x^2 + \frac{4000}{x} \rightarrow$

$S' = 2x - \frac{4000}{x^2} = \frac{2x^3 - 4000}{x^2} = 0 \rightarrow 2x^3 - 4000 = 0 \rightarrow$

$x = (2000)^{1/3} = 10 \cdot 2^{1/3}$;

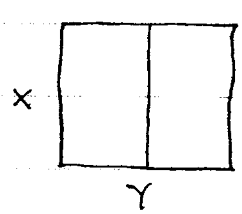


$x = 10 \cdot 2^{1/3} \approx 12.6$ in.

$y = 5 \cdot 2^{1/3} \approx 6.3$ in.

$S = 300 \cdot 2^{2/3} \approx 476$ in.^{2}

$\boxed{216:15}$

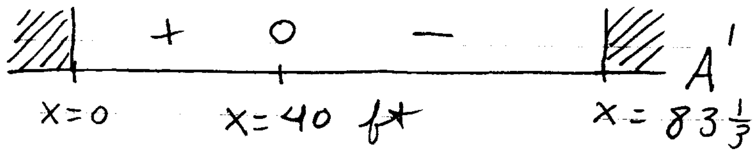


$3x + 2y = 240 \rightarrow y = \frac{240 - 3x}{2}$,

maximize area

$$A = XY = X\left(\frac{240-3X}{2}\right) = 120X - \frac{3}{2}X^2 \rightarrow$$

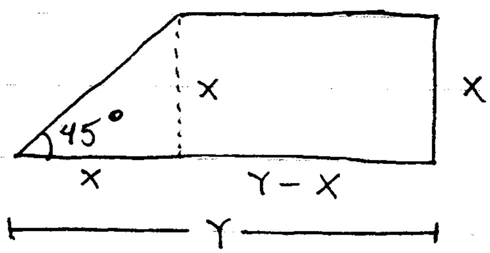
$$A' = 120 - 3X = 0 \rightarrow X = 40 \;;$$



$$Y = 60 \text{ ft}$$

$$A = 2400 \text{ ft.}^2$$

$\boxed{216:16}$



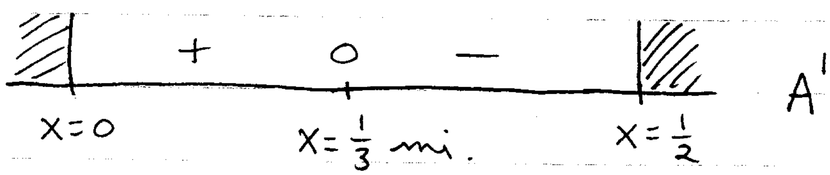
$X + Y = 1 \rightarrow Y = 1 - X,$

maximize area

$$A = \frac{1}{2}X^2 + X(Y-X) = \frac{1}{2}X^2 - X^2 + XY$$

$$= -\frac{1}{2}X^2 + X(1-X) = X - \frac{3}{2}X^2 \rightarrow$$

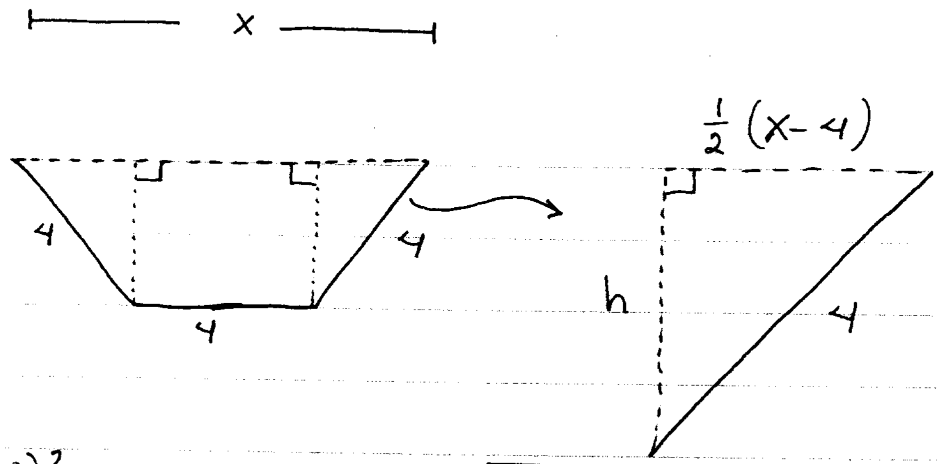
$$A' = 1 - 3X = 0 \rightarrow X = \frac{1}{3} \;;$$



$$Y = \frac{2}{3} \text{ mi.}$$

$$A = \frac{1}{6} \text{ mi.}^2$$

$\boxed{216:19}$



$h^2 + \left(\frac{1}{2}(x-4)\right)^2 = 4^2 \rightarrow \quad h = \sqrt{12 + 2x - \frac{1}{4}x^2}$,

maximize area

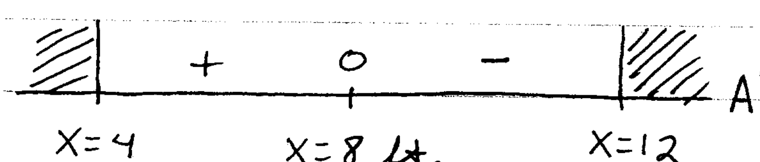
$$A = \frac{1}{2}(x+4)\sqrt{12+2x-\frac{1}{4}x^2} \longrightarrow$$

$$A' = \frac{1}{2}(x+4)\cdot\frac{1}{2}\left(12+2x-\frac{1}{4}x^2\right)^{-\frac{1}{2}}\cdot\left(2-\frac{1}{2}x\right) + \frac{1}{2}\left(12+2x-\frac{1}{4}x^2\right)^{\frac{1}{2}}$$

$$= \frac{\frac{1}{4}(x+4)\left(2-\frac{1}{2}x\right)}{\left(12+2x-\frac{1}{4}x^2\right)^{1/2}} + \frac{\frac{1}{2}\left(12+2x-\frac{1}{4}x^2\right)^{1/2}}{1}$$

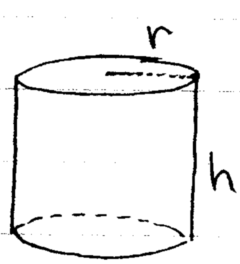
$$= \frac{-\frac{1}{4}x^2 + x + 8}{\left(12+2x-\frac{1}{4}x^2\right)^{1/2}} = 0 \longrightarrow -\frac{1}{4}\left(x^2 - 4x - 32\right) = 0 \rightarrow$$

$(x-8)(x+4) = 0 \rightarrow x = 8$;



$A = 12\sqrt{3} \approx 20.8$ ft.^{2}

$\boxed{216:21}$



$h + 2\pi r = 108 \rightarrow h = 108 - 2\pi r$,

maximize volume

$$V = \pi r^2 h = \pi r^2 (108 - 2\pi r) = 108\pi r^2 - 2\pi^2 r^3 \rightarrow$$

$$V' = 216\pi r - 6\pi^2 r^2 = 6\pi r(36 - \pi r) = 0 \rightarrow$$

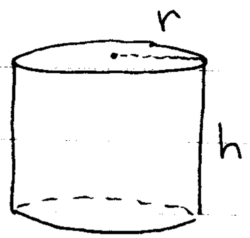
$r = \cancel{0}$ or $\dfrac{36}{\pi}$;

$$\begin{array}{cccccc} & + & 0 & - & & V' \\ r=0 & & r = \frac{36}{\pi} \approx 11.5 \text{ in.} & & r = \frac{54}{\pi} & \end{array}$$

$h = 36$ in.

$V = \dfrac{46{,}656}{\pi} \approx 14{,}851$ in.^{3}

$\boxed{216:22}$



$h + 2\pi r = 108 \rightarrow h = 108 - 2\pi r$,

maximize surface area

$$S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r(108 - 2\pi r)$$

$$= 2\pi r^2 + 216\pi r - 4\pi^2 r^2 \rightarrow$$

$$S' = 4\pi r + 216\pi - 8\pi^2 r = 0$$

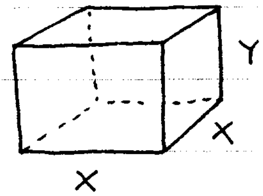
$r = \dfrac{-216\pi}{4\pi - 8\pi^2} = \dfrac{54}{2\pi - 1}$;

$$\begin{array}{cccccc} & + & 0 & - & & S' \\ r=0 & & r = \frac{54}{2\pi-1} \approx 10.2 \text{ in.} & & r = \frac{54}{\pi} & \end{array}$$

$h = \dfrac{108\pi - 108}{2\pi - 1} \approx 43.8$ in.

$S = 3468$ in.^{2}

$\boxed{216:27}$



$X^2 Y = 100 \rightarrow Y = \dfrac{100}{X^2}$,

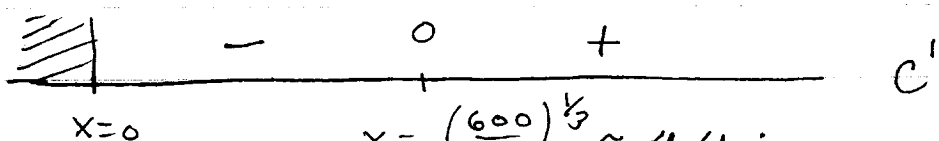
minimize cost

$C = 2(X^2) + 5(X^2) + 3(4XY)$

$= 7X^2 + 12XY = 7X^2 + 12X\left(\dfrac{100}{X^2}\right) = 7X^2 + \dfrac{1200}{X} \rightarrow$

$C' = 14X - \dfrac{1200}{X^2} = \dfrac{14X^3 - 1200}{X^2} = 0 \rightarrow$

$14X^3 - 1200 = 0 \rightarrow X = \left(\dfrac{600}{7}\right)^{1/3}$;



$X = \left(\dfrac{600}{7}\right)^{1/3} \approx 4.4$ in.

$Y = \left(\dfrac{7}{6}\right)^{2/3} \cdot 100^{1/3} \approx 5.1$ in.

$C = \$4.08$