HW #17

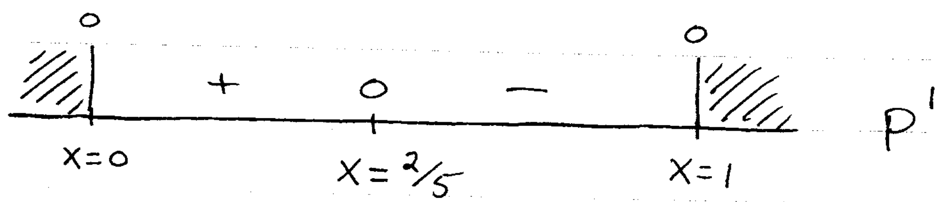
Section 4.7

$\boxed{216:18}$ $x \geq 0,\ y \geq 0,\quad x + y = 1 \to y = 1 - x,$

maximize $P = x^2 y^3 = x^2 (1-x)^3 \to$

$$P' = x^2 \cdot 3(1-x)^2 \cdot (-1) + 2x \cdot (1-x)^3$$

$$= x(1-x)^2 \cdot [-3x + 2(1-x)] = x(1-x)^2 \cdot [2 - 5x] = 0$$

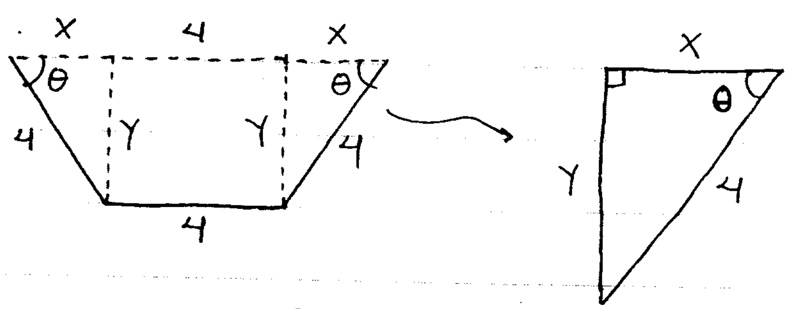


$x = 2/5$

$y = 3/5$

$P = \frac{108}{3125}$

$\boxed{216:20a}$



$\cos\theta = \frac{x}{4} \to x = 4\cos\theta,$

$\sin\theta = \frac{y}{4} \to y = 4\sin\theta,$ so maximize area

$$A = \tfrac{1}{2}(4 + (4 + 2x)) \cdot y = (4 + x)\,y = (4 + 4\cos\theta)(4\sin\theta)$$

$$= 16(1 + \cos\theta)\sin\theta \to$$

$$A' = 16(1+\cos\theta)\cdot\cos\theta + -16\sin\theta \cdot \sin\theta$$

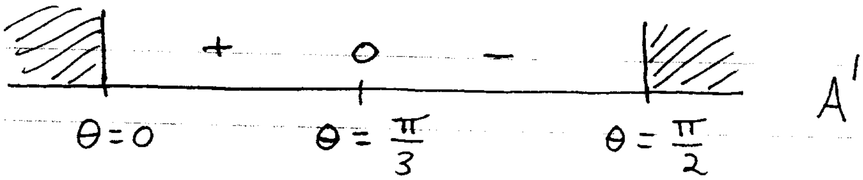
$$= 16\cos\theta + 16\cos^2\theta - 16(1 - \cos^2\theta)$$

$$= 16 \cdot [2\cos^2\theta + \cos\theta - 1]$$

$$= 16(2\cos\theta - 1)(\cos\theta + 1) = 0 \to$$

$\cos\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}$ or

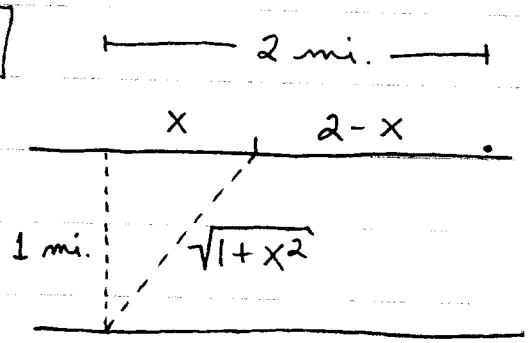
$\cos\theta = -1 \rightarrow \theta = \pi$ (No!)



$x = 2$ ft.

$y = 2\sqrt{3}$ ft., $A = 12\sqrt{3}$ ft.^{2}

$\boxed{216:30}$



land : \$3(5280)/mi.

water : \$5(5280)/mi

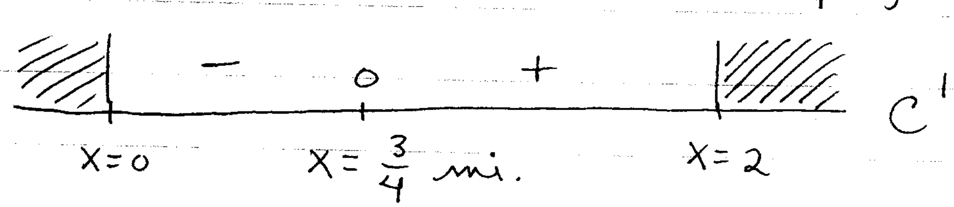
minimize cost

$$C = C_{water} + C_{land} = \left(5\sqrt{1+x^2} + 3(2-x)\right)(5280) \rightarrow$$

$$C' = \left(5 \cdot \tfrac{1}{\cancel{2}}(1+x^2)^{-1/2} \cdot (\cancel{2}x) - 3\right)(5280)$$

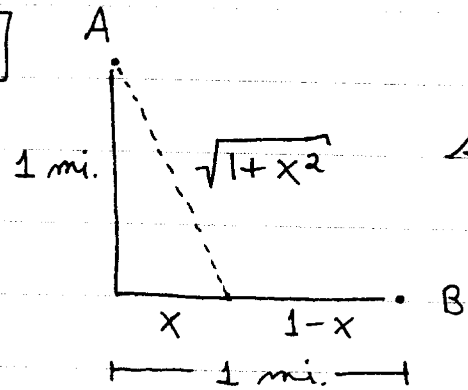
$$= \left(\frac{5x}{\sqrt{1+x^2}} - 3\right)(5280) = 0 \rightarrow 5x = 3\sqrt{1+x^2} \rightarrow$$

$$25x^2 = 9(1+x^2) \rightarrow 16x^2 = 9 \rightarrow x = \frac{3}{4};$$



$C = \$52,800$

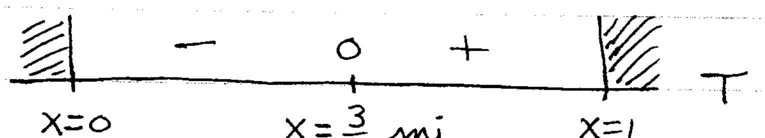
$\boxed{216:41\text{ c}}$



grass : 3 mph
sidewalk : 5 mph

minimize time

$$T = T_{grass} + T_{sidewalk}$$
$$= \frac{\sqrt{1+x^2}}{3} + \frac{1-x}{5} \rightarrow$$
$$T' = \frac{1}{3}\cdot\frac{1}{\cancel{2}}(1+x^2)^{-1/2}\cdot(\cancel{2}x) - \frac{1}{5} = \frac{x}{3\sqrt{1+x^2}} - \frac{1}{5} = 0 \rightarrow$$
$$\frac{x}{3\sqrt{1+x^2}} = \frac{1}{5} \rightarrow 5x = 3\sqrt{1+x^2} \rightarrow 25x^2 = 9(1+x^2) \rightarrow$$
$$16x^2 = 9 \rightarrow x = \frac{3}{4};$$



$T = .47$ hr.

$\boxed{216:47}$ S : mph and D : vehicles/mi. and

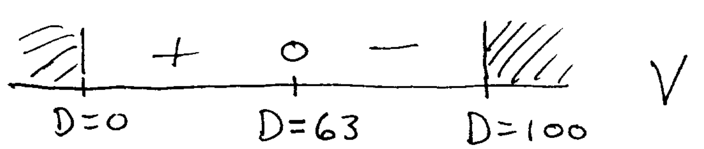
$S = 42 - \frac{D}{3}$ for $D \le 100$.

a.) vehicles per hour is (check units!)

$$V = DS = 42D - \frac{1}{3}D^2.$$

b.) maximize V :

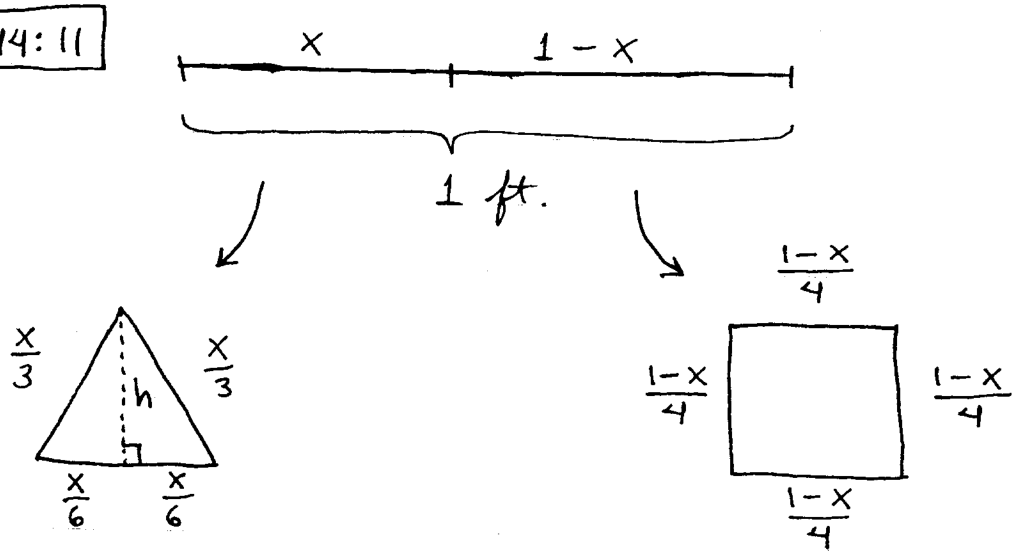
$$V' = 42 - \frac{2}{3}D = 0 \longrightarrow D = 63 \text{ vehicles/mi.}$$



$S = 21$ mph, $V = 1323$ vehicles/hr.

Review Section

$\boxed{244:11}$



$h^2 + \left(\frac{x}{6}\right)^2 = \left(\frac{x}{3}\right)^2 \rightarrow h = \frac{1}{2\sqrt{3}}x$; *maximize (minimize)*

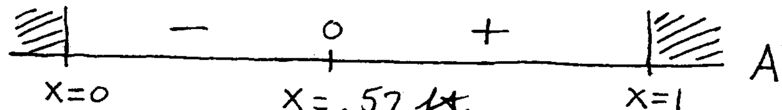
total area $A = A_\Delta + A_\square \rightarrow$

$$A = \left(\frac{x}{6}\right)h + \left(\frac{1-x}{4}\right)^2 = \frac{x}{6}\cdot\frac{1}{2\sqrt{3}}x + \frac{1}{16}(1-x)^2$$

$$= \frac{1}{12\sqrt{3}}x^2 + \frac{1}{16}(1-x)^2 \longrightarrow$$

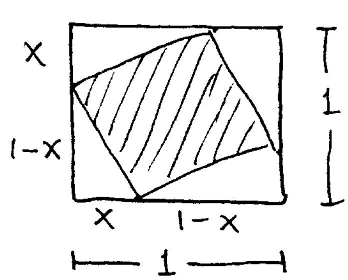
$$A' = \frac{1}{6\sqrt{3}}x + \frac{1}{8}(1-x)(-1) = \frac{1}{6\sqrt{3}}x - \frac{1}{8} + \frac{1}{8}x = 0 \longrightarrow$$

$$x = \frac{\frac{1}{8}}{\frac{1}{6\sqrt{3}} + \frac{1}{8}} \approx .57;$$



$\underbrace{A = .0625\text{ ft.}^2}_{\text{max.}}$ $\underbrace{A = .0272\text{ ft.}^2}_{\text{min.}}$ $A = .0481\text{ ft.}^2$

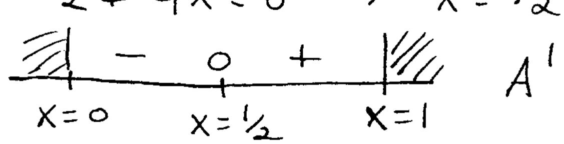
$\boxed{244:46}$



minimize area of shaded square:

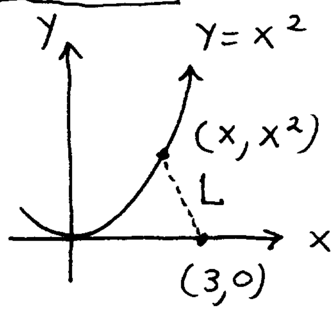
$$A = 1 - 4\cdot\tfrac{1}{2}x(1-x) = 1 - 2x + 2x^2 \rightarrow$$

$$A' = -2 + 4x = 0 \rightarrow x = \tfrac{1}{2}$$



$A = \frac{1}{2}$

$\boxed{244:47}$ *minimize length*



$$L = \sqrt{(x-3)^2 + (x^2-0)^2}$$
$$= \sqrt{(x-3)^2 + x^4} \longrightarrow$$

$$L' = \tfrac{1}{2}\left((x-3)^2 + x^4\right)^{-1/2} \cdot \left[2(x-3) + 4x^3\right]$$
$$= \frac{x-3+2x^3}{\sqrt{(x-3)^2 + x^4}} = 0 \longrightarrow \quad x-3+2x^3 = 0 \longrightarrow$$

$x = 1$;

$-$ 0 $+$ L'

$x = 1$

$L = \sqrt{5}$

$\boxed{244:68}$ $x, y > 0$, $xy = 2 \rightarrow y = \frac{2}{x}$,

minimize sum of squares

$$S = x^2 + y^2 = x^2 + \left(\frac{2}{x}\right)^2 = x^2 + \frac{4}{x^2} \longrightarrow$$
$$S' = 2x - \frac{8}{x^3} = \frac{2x^4 - 8}{x^3} = 0 \rightarrow 2x^4 - 8 = 0 \longrightarrow$$

$x = 4^{1/4}$;

$-$ 0 $+$ S'

$x = 0$ $x = 4^{1/4} = \sqrt{2}$

$y = \sqrt{2}$

$S = 4$