HW # 18

-30

Section 4.8

$\boxed{226:5}$ $\frac{2}{\pi}XY + \sin Y = 2 \rightarrow$

$\frac{2}{\pi}X \cdot Y' + \frac{2}{\pi} \cdot Y + \cos Y \cdot Y' = 0 \rightarrow \left(\frac{2}{\pi}X + \cos Y\right)Y' = \frac{-2}{\pi}Y \rightarrow$

$Y' = \dfrac{-\frac{2}{\pi}Y}{\frac{2}{\pi}X + \cos Y}$, at $\left(1, \frac{\pi}{2}\right) \rightarrow Y' = \dfrac{-1}{\frac{2}{\pi} + 0} = \dfrac{-\pi}{2}$.

$\boxed{226:6}$ $2Y^3 + 4XY + X^2 = 7 \rightarrow$

$6Y^2 Y' + 4XY' + 4Y + 2X = 0 \rightarrow$

$(6Y^2 + 4X)Y' = -2X - 4Y \rightarrow Y' = \dfrac{-2X - 4Y}{4X + 6Y^2} = \dfrac{-X - 2Y}{2X + 3Y^2}$,

at $(1,1) \rightarrow Y' = \dfrac{-1-2}{2+3} = \dfrac{-3}{5}$.

$\boxed{226:8}$ $X + \tan(XY) = 2 \rightarrow$

$1 + \sec^2(XY) \cdot [XY' + Y] = 0 \rightarrow 1 + X\sec^2(XY) \cdot Y' + \sec^2(XY) \cdot Y = 0 \rightarrow$

$Y' = \dfrac{-1 - Y\sec^2(XY)}{X\sec^2(XY)}$, at $\left(1, \frac{\pi}{4}\right) \rightarrow$

$Y' = \dfrac{-1 - \frac{\pi}{4}(\sqrt{2})^2}{1 \cdot (\sqrt{2})^2} = \dfrac{-1}{2} - \dfrac{\pi}{4}$.

$\boxed{226:20}$ $\sin^3(XY) + \cos(X+Y) + X = 1 \rightarrow$

$3\sin^2(XY) \cdot \cos(XY) \cdot [XY' + Y] - \sin(X+Y) \cdot [1 + Y'] + 1 = 0 \rightarrow$

$3\sin^2(XY) \cdot \cos(XY) \cdot XY' + 3\sin^2(XY) \cdot \cos(XY) \cdot Y$

$- \sin(X+Y) - \sin(X+Y) \cdot Y' + 1 = 0 \rightarrow$

$$Y' = \frac{\sin(X+Y) - 1 - 3Y\sin^2(XY)\cos(XY)}{3X\sin^2(XY) \cdot \cos(XY) - \sin(X+Y)}.$$

$\boxed{226:22}$ $x^5 + xy + y^5 = 35 \rightarrow$

$5x^4 + xy' + y + 5y^4 y' = 0 \rightarrow$

$y' = \dfrac{-5x^4 - y}{x + 5y^4}$, at $(1,2) \rightarrow y' = \dfrac{-7}{81}$ and

$y'' = \dfrac{(x+5y^4)(-20x^3 - y') - (-5x^4 - y)\cdot(1 + 20y^3 y')}{(x+5y^4)^2}$,

let $x=1$, $y=2$, $y' = \dfrac{-7}{81}$ then

$$y'' = \frac{(81)\left(\frac{-1613}{81}\right) + (7)\left(\frac{-1039}{81}\right)}{(81)^2} = \frac{-137,926}{531,441}.$$

$\boxed{226:26}$ $x^3 + xy^2 + x^3 y^5 = 3 \rightarrow$

$3x^2 + x\cdot 2yy' + y^2 + 3x^2 y^5 + x^3 \cdot 5y^4 y' = 0 \rightarrow$

$y' = \dfrac{-3x^2 - y^2 - 3x^2 y^5}{2xy + 5x^3 y^4}$ at $(1,1) \rightarrow$

$y' = \dfrac{-7}{7} = -1$ so tangent line at $(1,1)$ is

$y - 1 = -1(x-1)$ or $\boxed{y = 2 - x}$; this line does not pass through the point $(-2,3)$.

$\boxed{226:25}$ $\boxed{x^2+xy+y^2=12}$ $\xrightarrow{D}$

$2x+xy'+y+2yy'=0 \rightarrow (x+2y)y'=-2x-y \rightarrow$

$\boxed{y'=\dfrac{-2x-y}{x+2y}}=0 \rightarrow -2x-y=0 \rightarrow \boxed{y=-2x}$ (substitute into original equation) $\rightarrow x^2-2x^2+4x^2=12 \rightarrow$

$3x^2=12 \rightarrow x=\pm 2$; if $x=2,\ y=-4$ and

if $x=-2,\ y=4$; use Second Derivative Test :

$$y''=\frac{(x+2y)(-2-y')-(-2x-y)(1+2y')}{(x+2y)^2}$$ then

$x=2,\ y=-4,\ y'=0 \rightarrow y''=\frac{1}{3}>0$ so this point determines a minimum value ;

$x=-2,\ y=4,\ y'=0 \rightarrow y''=\frac{-1}{3}$ so this point determines a maximum value .

The graph of $x^2+xy+y^2=12$ is below :