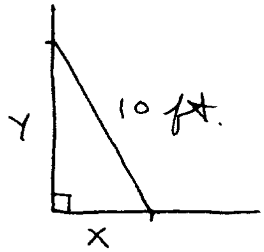
HW #19

Section 4.4

$\boxed{196:3}$



$\frac{dx}{dt} = 1$ ft./sec. ,

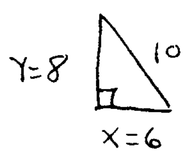
$x^2 + Y^2 = 10^2$,

Find $\frac{dY}{dt}$ when $x = 6$ ft. , $x = 8$ ft. , $x = 9$ ft. :

$x^2 + Y^2 = 100 \rightarrow \cancel{2}x \frac{dx}{dt} + \cancel{2}Y \frac{dY}{dt} = 0 \rightarrow$

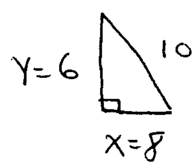
$\frac{dY}{dt} = \frac{-x \frac{dx}{dt}}{Y}$;

a.)



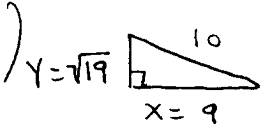
$\frac{dY}{dt} = \frac{-6 \cdot (1)}{8} = \frac{-3}{4}$ ft./sec.

b.)



$\frac{dY}{dt} = \frac{-8 \cdot (1)}{6} = \frac{-4}{3}$ ft./sec.

c.)



$\frac{dY}{dt} = \frac{-9 \cdot (1)}{\sqrt{19}} = \frac{-9}{\sqrt{19}}$ ft./sec.

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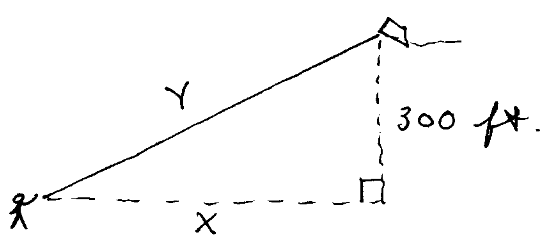
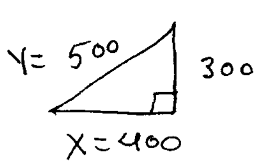


Find $\frac{dx}{dt}$ when $Y = 500$ ft. and $\frac{dY}{dt} = 20$ ft./sec. ;

$x^2 + 300^2 = Y^2 \rightarrow 2x \frac{dx}{dt} + 0 = 2Y \frac{dY}{dt} \rightarrow$

$\frac{dx}{dt} = \frac{Y \frac{dY}{dt}}{x} = \frac{(500)(20)}{400} = 25$ ft./sec.



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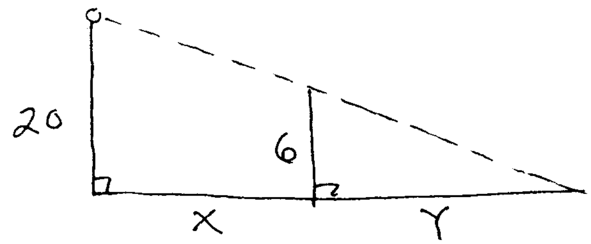

$\frac{dX}{dt} = 5$ ft./sec.,

find $\frac{dY}{dt}$;

by similar triangles

$$\frac{20}{X+Y} = \frac{6}{Y} \rightarrow 20Y = 6X + 6Y \rightarrow 7Y = 3X \rightarrow$$

$$Y = \frac{3}{7}X \rightarrow \frac{dY}{dt} = \frac{3}{7} \cdot \frac{dX}{dt} = \frac{3}{7} \cdot (5) = \frac{15}{7} \text{ ft./sec.},$$

no matter the distance from the lamp.

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$\frac{dV}{dt} = 100$ ft.^{3}/min.,

find $\frac{dr}{dt}$;

$$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

a.) $r = 10$ ft. $\rightarrow 100 = 4\pi(10)^2 \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{1}{4\pi}$ ft./min.

b.) $r = 20$ ft. $\rightarrow 100 = 4\pi(20)^2 \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{1}{16\pi}$ ft./min.

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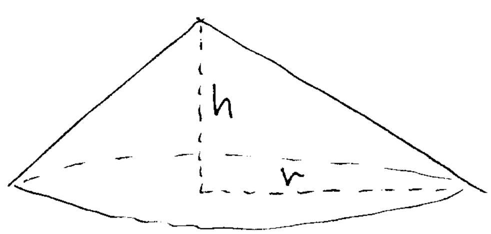

$\frac{dV}{dt} = 1000$ yd.^{3}/hr.,

$h = r$ and

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3$,

find $\frac{dh}{dt}$;

$$V = \frac{1}{3}\pi h^3 \rightarrow \frac{dV}{dt} = \pi h^2 \cdot \frac{dh}{dt}$$

a.) $h = 20$ yd. $\rightarrow 1000 = \pi(20)^2 \cdot \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{5}{2\pi}$ yd./hr.

b.) $h = 100$ yd. $\rightarrow 1000 = \pi(100)^2 \cdot \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{1}{10\pi}$ yd./hr.

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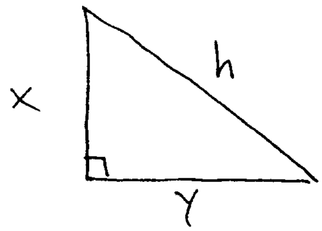
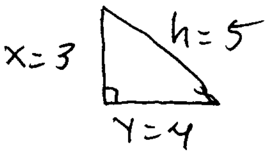


$\frac{dx}{dt} = 5$ ft./sec.,

$\frac{dY}{dt} = -6$ ft./sec.,

find $\frac{dh}{dt}$ when $X = 3$ ft., $Y = 4$ ft. ;

$$X^2 + Y^2 = h^2 \rightarrow \cancel{2}\underset{\underset{3}{\uparrow}}{X} \cdot \underset{\underset{5}{\uparrow}}{\frac{dx}{dt}} + \cancel{2}\underset{\underset{4}{\uparrow}}{Y} \underset{\underset{-6}{\uparrow}}{\frac{dY}{dt}} = \cancel{2}\underset{\underset{5}{\uparrow}}{h} \frac{dh}{dt}$$



$$\rightarrow 15 - 24 = 5 \frac{dh}{dt}$$

$$\rightarrow \frac{dh}{dt} = \frac{-9}{5} \text{ ft./sec.}$$, so

h is decreasing.

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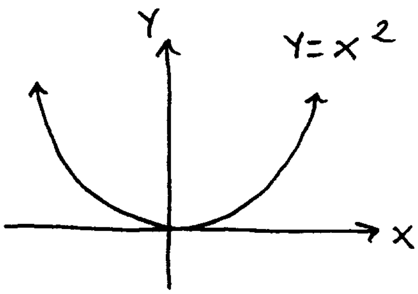

$\dot{x} = \frac{dx}{dt} = 3$ (assume x and Y are functions of t.)

$$Y = x^2 \xrightarrow{D_t} \dot{Y} = 2x \cdot \dot{x} \rightarrow$$

$$\dot{Y} = 2x(3) \rightarrow \boxed{\dot{Y} = 6x} ;$$

$$\ddot{Y} = 6\dot{x} = 6(3) = 18 \longrightarrow \boxed{\ddot{Y} = 18}$$

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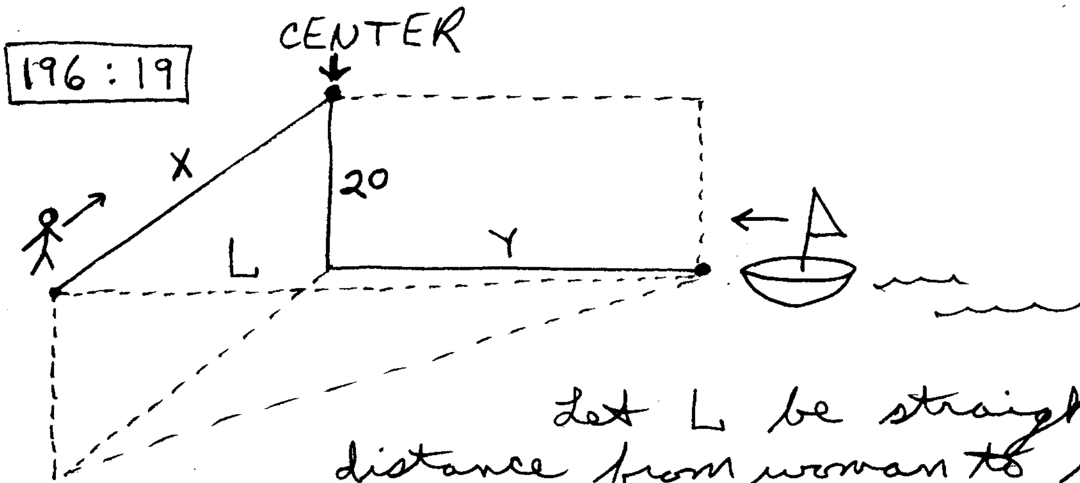
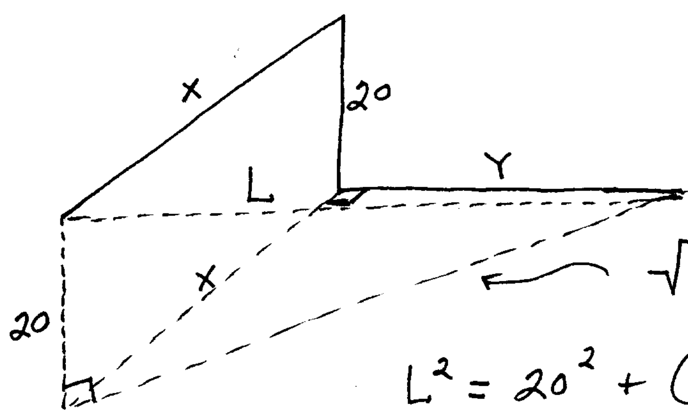


Let L be straight line distance from woman to boat.

Given $\frac{dY}{dt} = -10$ ft./sec. (Y is decreasing)

and $\frac{dX}{dt} = -5$ ft./sec. (X is decreasing),

find $\frac{dL}{dt}$ when $X = 50$ ft. and $Y = 0$ ft. :



$\sqrt{X^2+Y^2}$ and

$$L^2 = 20^2 + \left(\sqrt{X^2+Y^2}\right)^2 \rightarrow$$

$$L = \sqrt{400 + X^2 + Y^2}$$ (assume X, Y, and L are functions of t.) $\rightarrow$

$$\frac{dL}{dt} = \frac{1}{2}(400 + X^2 + Y^2)^{-1/2} \cdot \left[2X \cdot \frac{dX}{dt} + 2Y \cdot \frac{dY}{dt}\right]$$

$$= \frac{1}{2}(400 + 2500 + 0)^{-1/2}\left[2(50)(-5) + 2(0)(-10)\right]$$

$$= \frac{-250}{\sqrt{2900}} = \frac{-25}{\sqrt{29}}$$ ft./sec. (so L is decreasing).

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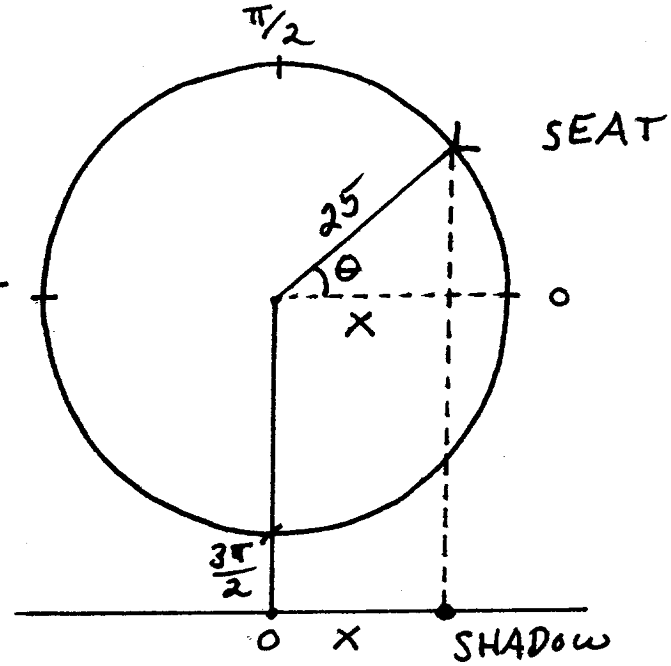


$$\frac{d\theta}{dt} = \left(\frac{\frac{1}{10}\text{ rev.}}{\text{sec.}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev.}}\right)$$
$$= \frac{\pi}{5} \text{ rad./sec.} ,$$

a.) find $\frac{dx}{dt}$ when $\theta = \frac{\pi}{6}$ (2 o'clock) :

$$\cos\theta = \frac{x}{25}$$ (assume θ and x are functions of t)

$$\xrightarrow{D_t} -\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{25} \cdot \frac{dx}{dt} \rightarrow$$

$$\frac{dx}{dt} = -25 \cdot \sin\theta \cdot \underbrace{\frac{d\theta}{dt}}_{\frac{\pi}{5}} \rightarrow \boxed{\frac{dx}{dt} = -5\pi \sin\theta} ;$$

if $\theta = \frac{\pi}{6}$ then $\frac{dx}{dt} = -5\pi \sin\frac{\pi}{6} = -5\pi\left(\frac{1}{2}\right)$ ft/sec.

b.) find $\frac{dx}{dt}$ when $\theta = \frac{\pi}{3}$ (1 o'clock) :

$$\frac{dx}{dt} = -5\pi \sin\left(\frac{\pi}{3}\right) = -5\pi \cdot \frac{\sqrt{3}}{2} \text{ ft./sec.}$$

c.) Find maximum and minimum SPEED (absolute value of velocity) :

velocity $\frac{dx}{dt} = -5\pi \sin\theta$ so speed is

speed $= \left|\frac{dx}{dt}\right| = 5\pi \left|\sin\theta\right|$, so speed is maximum when $\theta = \frac{\pi}{2}$ (top) or $\frac{3\pi}{2}$ (bottom)

and minimum when $\theta = 0$ (3 o'clock) or π (9 o'clock) .

Review Section

$\boxed{243:33}$ Find minimum slope of

$Y = x^3 - 9x^2 + 15x$: "slope" equation is

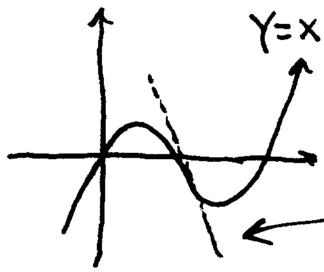
$Y' = 3x^2 - 18x + 15$, to find minimum

slope, take derivative of "slope"

equation and make a sign chart:

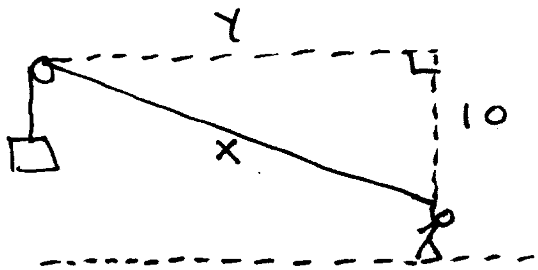
$Y'' = 6x - 18 = 0$ $\quad$ $-\ \ 0\ \ +$ $\quad Y''$

$x = 3$



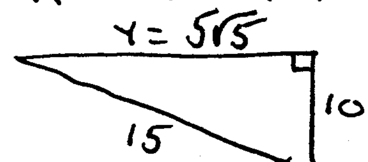
min slope $Y' = -12$

$\boxed{243:54}$



$Y^2 + 10^2 = x^2$ and $\frac{dY}{dt} = 5$ ft./sec., find

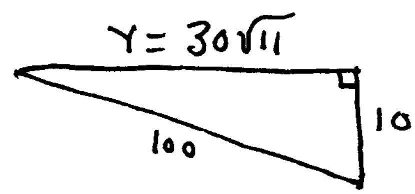
$\frac{dx}{dt}$ when a.) $x = 15$ ft. : $x^2 = 100 + Y^2$



$\xrightarrow{D_t}$ $\boxed{\cancel{2}x \cdot \frac{dx}{dt} = \cancel{2}Y \frac{dY}{dt}}$

$\longrightarrow (15)\frac{dx}{dt} = (5\sqrt{5})(5) \rightarrow \frac{dx}{dt} = \frac{25\sqrt{5}}{15} = \frac{5\sqrt{5}}{3}$ ft./sec.

b.) $x = 100$ ft :



$(100)\frac{dx}{dt} = (30\sqrt{11})(5) \rightarrow$

$\frac{dx}{dt} = \frac{15\cancel{0}\sqrt{11}}{10\cancel{0}} = \frac{3\sqrt{11}}{2}$ ft./sec.

Section 3.6

$\boxed{153:53}$ Temperature at x is $T = x^2$ and bug crawls at $\frac{dx}{dt} = 2$ cm./min. Find $\frac{dT}{dt}$ when $x = 3$ cm.

$$T = x^2 \rightarrow \frac{dT}{dt} = 2 \cdot x \cdot \frac{dx}{dt} = 2(3) \cdot (2) = 12 \text{ degrees/min.}$$