HW #2

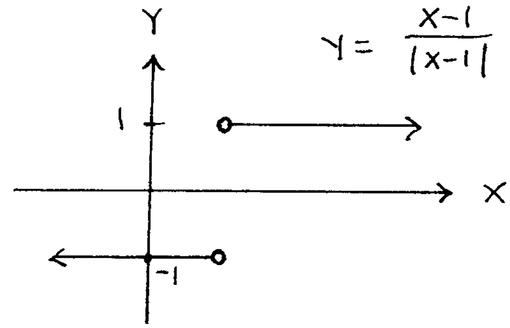
Section 2.3

$\boxed{34:4}$ $\lim_{x \to 3} \frac{x^2-9}{x-3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 6$

$\boxed{34:5}$ $\lim_{x \to 1} \frac{x^4-1}{x^3-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 1} \frac{(x^2-1)(x^2+1)}{(x-1)(x^2+x+1)}$

$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{(x-1)}\,(x^2+x+1)} = \frac{(2)(2)}{3} = \frac{4}{3}$

$\boxed{34:8}$ $\lim_{x \to 5} \frac{3x+5}{4x} = \frac{15+5}{20} = 1$



$\boxed{34:10}$ $\lim_{x \to 3} \pi^2 = \pi^2$

$\boxed{34:13}$ $\lim_{x \to 1^+} \frac{x-1}{|x-1|} \overset{x-1>0!}{=} \lim_{x \to 1^+} \frac{x-1}{x-1}$

(See graph) $= \lim_{x \to 1^+} 1 = 1$.

$\boxed{34:14}$ $\lim_{x \to 1^-} \frac{x-1}{|x-1|} \underset{x-1<0!}{=} \lim_{x \to 1^-} \frac{x-1}{-(x-1)} = \lim_{x \to 1^-} -1 = -1$

$\boxed{34:15}$ $\lim_{h \to 1} \frac{(1+h)^2-1}{h} = \frac{4-1}{1} = 3$

$\boxed{34:16}$ $\lim_{h \to 0} \frac{(1+h)^2-1}{h} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h \to 0} \frac{\cancel{1}+2h+h^2-\cancel{1}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2+h)}{\cancel{h}} = 2$

$\boxed{34:17}$ $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 2} \frac{2-x}{2x} \cdot \frac{1}{x-2}$

$= \lim_{x \to 2} \frac{-1}{2x} = \frac{-1}{4}$

$\boxed{34:20}$ $\lim_{x\to 1} \frac{3^x - 3}{2^x} = \frac{0}{2} = 0$

$\boxed{34:22}$ (a) $\lim_{x\to 1} f(x) = 2$ (b) $\lim_{x\to 2} f(x) = 2$

(c) $\lim_{x\to 3} f(x) = 1$ (d) $\lim_{x\to 4^-} f(x) = 2$

$\boxed{34:23}$

x	$\frac{3^x-1}{x}$
0.1	1.1612317
0.01	1.1046692
0.001	1.0992159
0.0001	1.098672
-0.0001	1.0985514

guess:
$\lim_{x\to 0} \frac{3^x-1}{x} \approx 1.098$

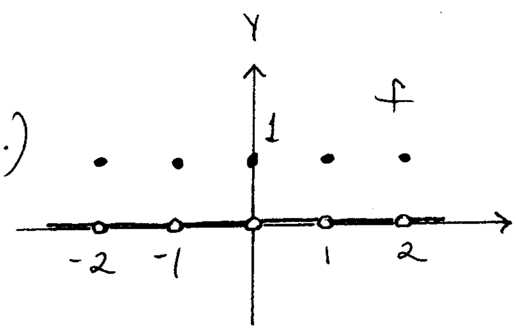
It will be shown later that the limit is $\ln 3$.

$\boxed{34:27}$ $\lim_{x\to 3} f(x) = 1$

$\boxed{34:28}$

$$f(x) = \begin{cases} 1, & x \text{ integer} \\ 0, & x \text{ not integer} \end{cases}$$

a.)



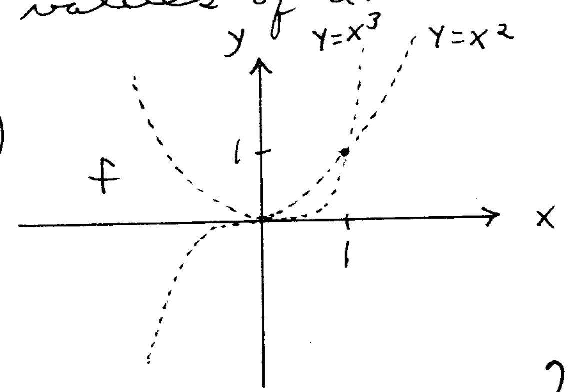
b.) $\lim_{x\to 3} f(x) = 0$

c.) $\lim_{x\to 3.5} f(x) = 0$ d.) $\lim_{x\to a} f(x)$ exists for all values of a.

$\boxed{34:31}$

$$f(x) = \begin{cases} x^2, & x \text{ rational} \\ x^3, & x \text{ irrational} \end{cases}$$

a.)



b.) $\lim_{x \to 2} f(x)$ does not exist.

c.) $\lim_{x \to 1} f(x) = 1$ d.) $\lim_{x \to 0} f(x) = 0$

e.) $\lim_{x \to a} f(x)$ exists for $a = 0$ and $a = 1$ only.

$\boxed{34:35}$ $f(n) = \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}$ (n+1 terms)

a.) $f(1) = 1 + \frac{1}{2} = \frac{3}{2} = 1.5$

$f(2) = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12} = 1.0833333$

$f(3) = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{57}{60} = 0.95$

$f(4) = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = 0.8845238$

$f(5) = \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} = 0.8456349$

$f(6) = \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} = 0.8198773$

$f(7) = \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} = 0.8015623$

$f(8) = \frac{1}{8} + \frac{1}{9} + \cdots + \frac{1}{16} = 0.7878718$

$f(9) = \frac{1}{9} + \frac{1}{10} + \cdots + \frac{1}{18} = 0.7772509$

$f(10) = \frac{1}{10} + \frac{1}{11} + \cdots + \frac{1}{20} = 0.7687714$

b) as n increases, f(n) decreases.

c.)d.) Since

$$\frac{1}{2n} + \frac{1}{2n} + \cdots + \frac{1}{2n} < \overbrace{\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n}}^{n+1 \text{ terms}} < \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n} \longrightarrow$$

$$\frac{n+1}{2n} < f(n) < \frac{n+1}{n} \longrightarrow$$

(if it exists!)

$$\lim_{n\to\infty} \frac{n+1}{2n} \le \lim_{n\to\infty} f(n) \le \lim_{n\to\infty} \frac{n+1}{n} \rightarrow$$

$$\lim_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) \le \lim_{n\to\infty} f(n) \le \lim_{n\to\infty} \left(1+\frac{1}{n}\right) \rightarrow$$

$$\frac{1}{2}+0 \le \lim_{n\to\infty} f(n) \le 1+0 \rightarrow$$

$$\frac{1}{2} \le \lim_{n\to\infty} f(n) \le 1 .$$

This conjecture follows from the succession of algebraically equivalent double inequalities.

Remark: It can be shown later that $\lim_{n\to\infty} f(n) = \ln 2$.