Section 4.6 $\qquad$ Newton : $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$\boxed{209:4}$ If $x^3 = a$, then x is the cube root of a ; let $f(x) = x^3 - a$ and use Newton's method to solve $f(x) = 0 \rightarrow$

$$f(x) = x^3 - a, \quad f'(x) = 3x^2 \rightarrow x_{n+1} = x_n - \frac{x_n^3 - a}{3x_n^2} \rightarrow$$

$$x_{n+1} = x_n - \left(\frac{x_n^3}{3x_n^2} - \frac{a}{3x_n^2} \right) = x_n - \left(\frac{1}{3} x_n - \frac{a}{3x_n^2} \right) \rightarrow$$

$$x_{n+1} = \frac{2}{3} x_n + \frac{a}{3x_n^2} \quad .$$

$\boxed{209:8}$ Estimate $\sqrt[3]{25} \rightarrow$ by $\boxed{209:4} \rightarrow$

$$x_{n+1} = \frac{2}{3} x_n + \frac{25}{3x_n^2} \quad \text{and} \quad x_1 = 3 \rightarrow$$

$$x_2 = \frac{2}{3} x_1 + \frac{25}{3x_1^2} = \frac{2}{3}(3) + \frac{25}{27} \approx 2.926 \ ,$$

$$x_3 = \frac{2}{3} x_2 + \frac{25}{3x_2^2} = \frac{2}{3}(2.926) + \frac{25}{3(2.926)^2} \approx 2.924$$

$\boxed{209:11}$ a.) Let $f(x) = x^5 + x - 1$, which is continuous for all values of x since f is a polynomial; and $f(0) = -1$ and $f(1) = 1$ and $m = 0$ is between $f(0)$ and $f(1)$, so by IMVT there is at least one number c, $0 < c < 1$, satisfying $f(c) = m$, i.e., $c^5 + c - 1 = 0$;

b.) $f'(x) = 5x^4 + 1$ so Newton's formula is

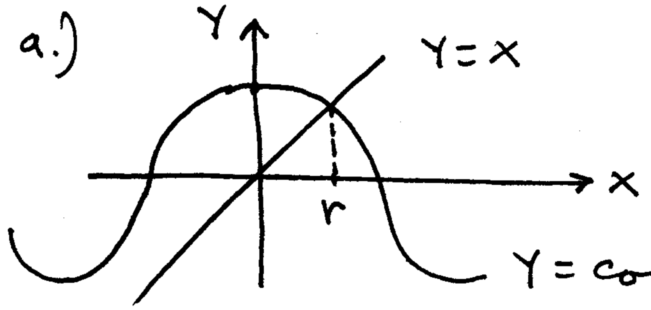
$$x_{n+1} = x_n - \frac{x_n^5 + x_n - 1}{5x_n^4 + 1} = \frac{5x_n^5 + \cancel{x_n} - x_n^5 - \cancel{x_n} + 1}{5x_n^4 + 1} \rightarrow$$

$$x_{n+1} = \frac{4x_n^5 + 1}{5x_n^4 + 1} \rightarrow \text{let } x_1 = \frac{1}{2} \text{ then}$$

$$x_2 = \frac{4x_1^5+1}{5x_1^4+1} = \frac{4\left(\frac{1}{32}\right)+1}{5\left(\frac{1}{16}\right)+1} = \frac{\frac{9}{8}}{\frac{21}{16}} = \frac{6}{7}$$

c.) Since $f'(x) = 5x^4+1 > 0$ for all x-values, f is a strictly increasing function. Thus, f crosses the x-axis only once and $f(x)=0$ has exactly one solution.

$\boxed{209:15}$ a.)



b.) By using a graphing calculator (TI 85) I estimate the root r to be $r \approx 0.7$

c.) $\cos x = x \rightarrow f(x) = \cos x - x = 0$ and $f'(x) = -\sin x - 1$ so Newton's formula is

$$x_{n+1} = x_n - \frac{\cos x_n - x_n}{-\sin x_n - 1} = \frac{-x_n \sin x_n - \cancel{x_n} - \cos x_n + \cancel{x_n}}{-\sin x_n - 1}$$

$$\rightarrow \quad x_{n+1} = \frac{x_n \sin x_n + \cos x_n}{1 + \sin x_n} \quad ; \text{ let } x_1 = 0.7$$

then $x_2 = \dfrac{x_1 \sin x_1 + \cos x_1}{1+\sin x_1} \approx 0.7394$,

$$x_3 = \frac{x_2 \sin x_2 + \cos x_2}{1+\sin x_2} \approx 0.7391,$$

$$x_4 = \frac{x_3 \sin x_3 + \cos x_3}{1+\sin x_3} \approx 0.7391 .$$

$\boxed{209:17}$ Let $f(x) = 2x^3 - 4x + 1$

a.) f is continuous for all x-values since f is a polynomial, and $m = 0$ is between $f(0) = 1$ and $f(1) = -1$. So by IMVT there is at least one number c, $0 \le c \le 1$, satisfying $f(c) = m$, i.e., $2c^3 - 4c + 1 = 0$

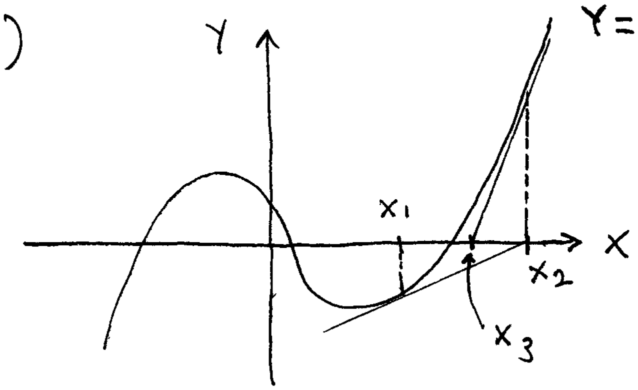
b.) $f'(x) = 6x^2 - 4$ so Newton's formula is

$$x_{n+1} = x_n - \frac{2x_n^3 - 4x_n + 1}{6x_n^2 - 4} = \frac{6x_n^3 - \cancel{4x_n} - 2x_n^3 + \cancel{4x_n} - 1}{6x_n^2 - 4} \rightarrow$$

$$x_{n+1} = \frac{4x_n^3 - 1}{6x_n^2 - 4} \text{ ; let } x_1 = 1 \text{ then}$$

$$x_2 = \frac{4x_1^3 - 1}{6x_1^2 - 4} = \frac{3}{2} \text{ and } x_3 = \frac{4x_2^3 - 1}{6x_2^2 - 4} \approx 1.316$$

c.)



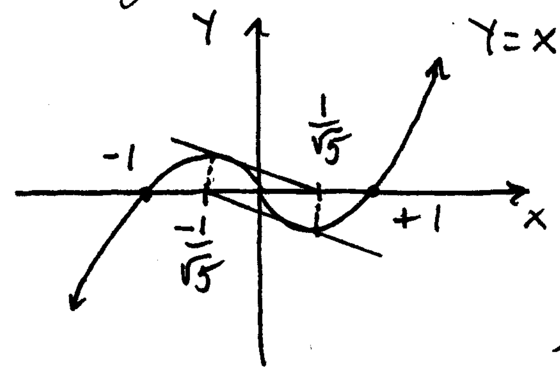
$\boxed{209:19}$ a.) $f(x) = x^3 - x$, $f'(x) = 3x^2 - 1$ so Newton's formula is

$$x_{n+1} = x_n - \frac{x_n^3 - x_n}{3x_n^2 - 1} = \frac{3x_n^3 - \cancel{x_n} - x_n^3 + \cancel{x_n}}{3x_n^2 - 1} = \frac{2x_n^3}{3x_n^2 - 1} \text{ ;}$$

let $x_1 = \frac{1}{\sqrt{5}}$ then $x_2 = \frac{2x_1^3}{3x_1^2 - 1} = \frac{\frac{2}{5\sqrt{5}}}{-\frac{2}{5}} = \frac{-1}{\sqrt{5}}$ and

$$x_3 = \frac{2x_2^3}{3x_2^2-1} = \frac{\frac{-2}{5\sqrt{5}}}{\frac{3}{5}-1} = \frac{+1}{\sqrt{5}} \; (!)$$ and so numbers simply repeat.

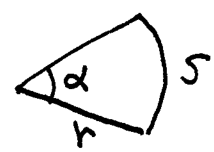
b.)



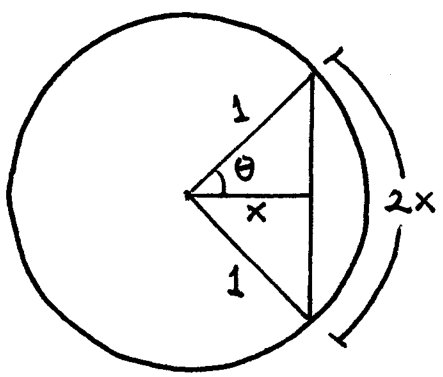
The slopes of the tangent lines at $x = \frac{1}{\sqrt{5}}$ and $x = \frac{-1}{\sqrt{5}}$ are leading to a useless, repeating succession of "guesses" to the roots of the polynomial.

$\boxed{209:20}$

arclength on circular arc is



$s = r\alpha$,



so here $2x = (1)(2\theta)$ or $\boxed{x = \theta}$; in addition, if $0 < \theta < \frac{\pi}{2}$, $\cos\theta = \frac{x}{1}$ or

$\boxed{x = \cos\theta}$; we need to solve $\cos\theta = \theta$ or $\cos\theta - \theta = 0$

(SEE $\boxed{209:15}$) ... so $\theta \approx 0.7391$ rad.

There are two solutions ($\theta \approx 0.7391$ and $\theta \approx \pi - 0.7391$) if we reflect our diagram into the 2nd and 3rd quadrants.)