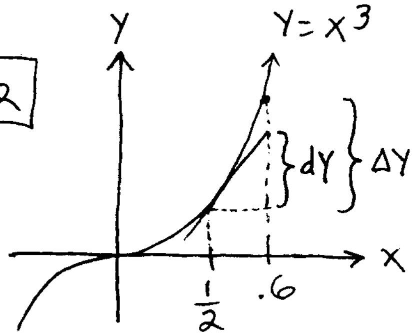
HW #21

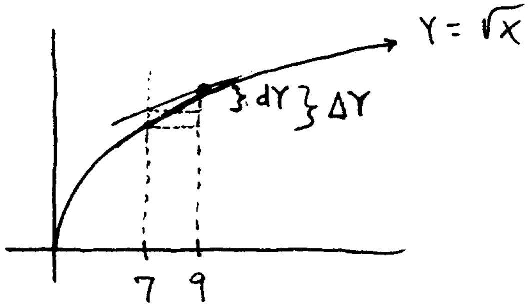
Section 4.9

$\boxed{232:2}$



$x: .5 \rightarrow .6$, $dx = .1$,
$\Delta Y = Y(.6) - Y(.5) = 0.091$,
$Y' = 3x^2$ and
$dY = Y'(.5) \cdot dx = 0.075$.

$\boxed{232:3}$



$x: 9 \rightarrow 7$, $dx = -2$,
$\Delta Y = Y(7) - Y(9) = -0.354$,
$Y' = \frac{1}{2\sqrt{x}}$ so

$dY = Y'(9) \cdot dx = -0.333$

$\boxed{232:7}$ $x: 100 \rightarrow 98$, $dx = -2$, $Y = \sqrt{x}$, $Y' = \frac{1}{2\sqrt{x}} \rightarrow$
$\Delta Y = Y(98) - Y(100) = \sqrt{98} - 10$ and
$dY = Y'(100) \cdot dx = -0.1 \rightarrow \Delta Y \approx dY$ so
$\sqrt{98} - 10 \approx -0.1 \rightarrow \sqrt{98} \approx 9.9$.

$\boxed{232:12}$ $x: 27 \rightarrow 28$, $dx = +1$, $Y = x^{1/3}$, $Y' = \frac{1}{3x^{2/3}} \rightarrow$
$\Delta Y = Y(28) - Y(27) = 28^{1/3} - 3$ and
$dY = Y'(27) \cdot dx = \frac{1}{27} \rightarrow \Delta Y \approx dY$ so
$28^{1/3} - 3 \approx \frac{1}{27} \rightarrow 28^{1/3} \approx 3\frac{1}{27}$

$\boxed{232:13}$ $x: \frac{\pi}{4} \rightarrow \frac{\pi}{4} - 0.01$, $dx = -0.01$,
$Y = \tan x$, $Y' = \sec^2 x \rightarrow$
$\Delta Y = Y\left(\frac{\pi}{4} - 0.01\right) - Y\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4} - 0.01\right) - 1$,

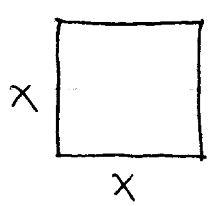
$$dY = Y'\left(\frac{\pi}{4}\right)\cdot dx = \sec^2\left(\frac{\pi}{4}\right)\cdot(-0.01)$$
$$= (\sqrt{2})^2(-0.01) = -0.02 \quad \text{and } \Delta Y \approx dY \text{ so}$$
$$\tan\left(\frac{\pi}{4}-0.01\right) - 1 \approx -0.02 \to \tan\left(\frac{\pi}{4}-0.01\right) \approx 0.98$$

$\boxed{232:18}$ $x: 0 \to 0.3$, $dx = +0.3$, $Y = \sin x$, $Y' = \cos x \to$
$\Delta Y = Y(0.3) - Y(0) = \sin(0.3)$ and
$dY = Y'(0)\cdot dx = +0.3 \to \Delta Y \approx dY$ so
$\sin(0.3) \approx 0.3$.

$\boxed{232:25}$ Let $f(x) = \frac{1}{x}$ and $x: 1 \to 1+h$ then $\Delta x = h$
$f'(x) = \frac{-1}{x^2}$ and $\Delta f \approx df \to$
$$f(1+h) - f(1) \approx f'(1)\cdot \Delta x \to \frac{1}{1+h} - 1 \approx -h \to$$
$$\frac{1}{1+h} \approx 1 - h.$$

$\boxed{232:28}$ Let $f(x) = x^{1/3}$ and $x: 1 \to 1+h$ then $\Delta x = h$
$f'(x) = \frac{1}{3}x^{-2/3}$ and $\Delta f \approx df \to$
$$f(1+h) - f(1) \approx f'(1)\cdot\Delta x \to (1+h)^{1/3} - 1 \approx \frac{1}{3}h \to$$
$$(1+h)^{1/3} \approx 1 + \frac{1}{3}h.$$

$\boxed{232:42}$



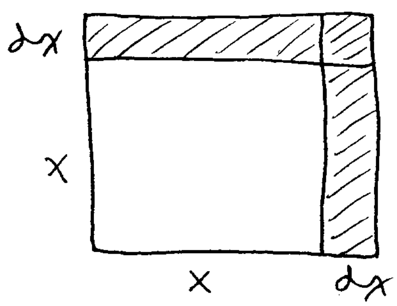
$$\frac{|dx|}{x} \le 5\%,$$

$$A = x^2 \text{ so } \frac{|\Delta A|}{A} \approx \frac{|dA|}{A} = \frac{|A'dx|}{A}$$
$$= \frac{|2x\,dx|}{x^2} = 2\frac{|dx|}{x} \le 10\%.$$

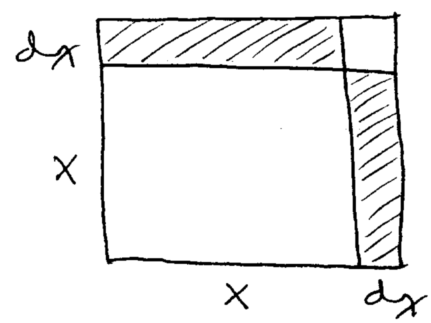
$\boxed{232:43}$ a.) x changes to $x + dx$ so

$$\Delta f = f(x+dx) - f(x) = (x+dx)^2 - x^2$$
$$= (x^2 + 2x\,dx + (dx))^2 - x^2$$
$$= 2x\,dx + (dx)^2 \quad ; \quad f'(x) = 2x \text{ so}$$
$$df = f'(x)\,dx = 2x\,dx$$

b.)



c.)



$\boxed{232:48}$

x	1.0	1.1	1.2	1.3	1.4	1.5
$f'(x)$	0.7	0.5	0.4	0.2	0.3	0.4

$$\frac{f(1.6)-f(1.5)}{1.6-1.5} \approx f'(1.5) \rightarrow \quad f(1.6) \approx (0.1)(0.4) + f(1.5) = \boxed{3.25}$$

$$\frac{f(1.5)-f(1.4)}{1.5-1.4} \approx f'(1.4) \rightarrow \quad f(1.5) \approx (0.1)(0.3) + f(1.4) = 3.21$$

$$\frac{f(1.4)-f(1.3)}{1.4-1.3} \approx f'(1.3) \rightarrow \quad f(1.4) \approx (0.1)(0.2) + f(1.3) = 3.18$$

$$\frac{f(1.3)-f(1.2)}{1.3-1.2} \approx f'(1.2) \rightarrow \quad f(1.3) \approx (0.1)(0.4) + f(1.2) = 3.16$$

$$\frac{f(1.2)-f(1.1)}{1.2-1.1} \approx f'(1.1) \rightarrow \quad f(1.2) \approx (0.1)(0.5) + f(1.1) = 3.12$$

$$\frac{f(1.1)-f(1)}{1.1-1} \approx f'(1) \rightarrow \quad f(1.1) \approx (0.1)(0.7) + f(1) = 3.07$$

$\boxed{232:41}$ $T = k\sqrt{\ell}$, $\dfrac{|\Delta \ell|}{\ell} \le p\%$,

estimate $\dfrac{|\Delta T|}{T}$:

$$\frac{|\Delta T|}{T} \approx \frac{|dT|}{T} = \frac{|T' \cdot \Delta \ell|}{T} = \frac{\left|\cancel{k} \cdot \frac{1}{2\sqrt{\ell}} \cdot \Delta \ell\right|}{\cancel{k}\sqrt{\ell}}$$

$$= \frac{1}{2} \cdot \frac{|\Delta \ell|}{\ell} \le \frac{1}{2} p\% = \frac{p}{2}\% .$$

Review Section

$\boxed{244:34}$ b.) Let $f(x) = x^{1/3}$, $x: 1 \to 1+h^2$, $\Delta x = h^2$,
and $\Delta f = f(1+h^2) - f(1) = \sqrt[3]{1+h^2} - 1$,
$df = f'(1)\cdot\Delta x = \frac{1}{3}(1)^{-2/3}\cdot h^2 = \frac{1}{3}h^2$;
since $\Delta f \approx df \to \sqrt[3]{1+h^2} - 1 \approx \frac{1}{3}h^2$
or $\sqrt[3]{1+h^2} \approx 1 + \frac{1}{3}h^2$.

c.) Let $f(x) = \frac{1}{x^2}$, $x: 1 \to 1-h$, $\Delta x = -h$,
$f'(x) = \frac{-2}{x^3}$, and $\Delta f = f(1-h) - f(1)$
$= \frac{1}{(1-h)^2} - 1$, $df = f'(1)\cdot\Delta x = -2\cdot(-h) = 2h$;
since $\Delta f \approx df \to \frac{1}{(1-h)^2} - 1 \approx 2h$ or
$\frac{1}{(1-h)^2} \approx 1 + 2h$.

$\boxed{244:42}$ Let $f(x) = \sin x$, $x: \frac{\pi}{6} \to \frac{\pi}{6} + h$, $\Delta x = h$,
$f'(x) = \cos x$, and $\Delta f = f(\frac{\pi}{6}+h) - f(\frac{\pi}{6})$
$= \sin(\frac{\pi}{6}+h) - \sin\frac{\pi}{6} = \sin(\frac{\pi}{6}+h) - \frac{1}{2}$,
$df = f'(\frac{\pi}{6})\cdot\Delta x = \cos\frac{\pi}{6}\cdot h = \frac{\sqrt{3}}{2}h$; since $\Delta f \approx df$
$\to \sin(\frac{\pi}{6}+h) - \frac{1}{2} \approx \frac{\sqrt{3}}{2}h$ or
$\sin(\frac{\pi}{6}+h) \approx \frac{1}{2} + \frac{\sqrt{3}}{2}h$.