HW # 22

Section 6.1

$\boxed{326:1}$ a.) $\log_2 32 = 5$

b.) $\log_3 81 = 4$

c.) $\log_{10} 0.001 = -3$

d.) $\log_5 1 = 0$

e.) $\log_{1000} 10 = 1/3$

f.) $\log_{49} 7 = \frac{1}{2}$

$\boxed{326:5}$ a.) $2^x = 7$

b.) $5^s = 2$

c.) $3^{-1} = \frac{1}{3}$

d.) $7^2 = 49$

$\boxed{326:14}$ $10^{2x}\, 3^{2x} = 5 \rightarrow 30^{2x} = 5 \rightarrow$
$\ln 30^{2x} = \ln 5 \rightarrow 2x \cdot \ln 30 = \ln 5 \rightarrow$
$$x = \frac{\ln 5}{2 \ln 30}.$$

$\boxed{326:16}$ $\log_3 5 = a \rightarrow 3^a = 5 \rightarrow 3 = 5^{\frac{1}{a}} \rightarrow$

$\log_5 3 = 1/a$.

$\boxed{326:18}$ a.) $\log_{10} 2^{1/2} = \frac{1}{2} \cdot \log_{10} 2 \approx 0.15$

b.) $\log_{10} 0.5 = \log_{10} \frac{1}{2} = \log_{10} 1 - \log_{10} 2$

$= 0 - 0.30 = -0.30$

c.) $\log_{10} \frac{2}{3} = \log_{10} 2 - \log_{10} 3 = 0.30 - 0.48 = -0.18$

d.) $\log_{10} 3^{1/3} = \frac{1}{3} \cdot \log_{10} 3 = 0.16$

e.) $\log_{10} 18 = \log_{10} 2 \cdot 3^2 = \log_{10} 2 + 2 \cdot \log_{10} 3$

$= 0.30 + 2(0.48) = 1.26$

f.) $\log_{10} 12 = \log_{10} 2^2 \cdot 3 = 2 \log_{10} 2 + \log_{10} 3$

$= 2(0.30) + (0.48) = 1.08$

g.) $\log_{10} 0.75 = \log_{10} \frac{3}{4} = \log_{10} 3 - \log_{10} 2^2$

$= \log_{10} 3 - 2 \log_{10} 2 = (0.48) - 2(0.30) = -0.12$

h.) $\log_{10} 7.5 = \log_{10} \frac{3 \cdot 10}{2^2} = \log_{10} 3 + \log_{10} 10$

$- 2 \log_{10} 2 = (0.48) + (1) - (0.60) = 0.88$

i.) $\log_{10} \frac{1}{7.5} = \log_{10} 1 - \log_{10} 7.5$

$= 0 - (0.88) = -0.88$

j.) $\log_{10} 0.075 = \log_{10} \frac{1}{100}(7.5) = \log_{10} 10^{-2} \cdot (7.5)$

$= \log_{10} 10^{-2} + \log_{10} 7.5 = (-2) + (0.88) = -1.12$

k.) $\log_{10} 30 \cdot 2^{5/3} = \log_{10} 3 \cdot 10 \cdot 2^{5/3}$

$= \log_{10} 3 + \log_{10} 10 + \frac{5}{3} \log_{10} 2$

$= (0.48) + (1) + \frac{5}{3}(0.30) = 1.98$

l.) $\log_{10} \frac{9}{32} = \log_{10} \frac{3^2}{2^5} = \log_{10} 3^2 - \log_{10} 2^5$

$= 2 \cdot \log_{10} 3 - 5 \cdot \log_{10} 2 = 2(0.48) - 5(0.30) = -0.54$

$\boxed{326:29}$ $\log_2 [\log_2 (\log_2 2^{1024})]$

$= \log_2 [\log_2 1024] = \log_2 [\log_2 2^{10}] = \log_2 10$

$\boxed{326:30}$ Is $\log_2 (c+d)$ ever equal to

$\log_2 c + \log_2 d$? <u>YES</u>: If $c = 2$ and $d = 2$

then $\log_2 (2+2) = \log_2 4 = 2$

and

$\log_2 2 + \log_2 2 = 1 + 1 = 2$.

Section 6.2

$\boxed{334:3}$ $\lim_{t\to 0} (1+t)^{1000} = 1^{1000} = 1$

$\boxed{334:4}$ $\lim_{x\to\infty} 1.001^{x} = +\infty$

$\boxed{334:5}$ $\lim_{h\to 0} (1+3h)^{\frac{1}{4h}} = \lim_{h\to 0} \left[(1+(3h))^{\frac{1}{(3h)}}\right]^{\frac{3}{4}} = e^{3/4}$

$\boxed{334:6}$ $\lim_{h\to 0} (1-h)^{1/h} = \lim_{h\to 0} \left[(1+(-h))^{\frac{1}{(-h)}}\right]^{-1} = e^{-1}$

$\boxed{334:7}$ $\lim_{\Delta x\to 0} \left(1+\frac{\Delta x}{2x}\right)^{x/\Delta x} = \lim_{\Delta x\to 0} \left(1+\left(\frac{\Delta x}{2x}\right)\right)^{\frac{1}{\left(\frac{\Delta x}{2x}\right)}\frac{1}{2}} = e^{\frac{1}{2}}$

$\boxed{334:8}$ $\lim_{n\to\infty} \left(1+\frac{3}{n}\right)^{\frac{n}{2}} = \lim_{n\to\infty} \left[\left(1+\frac{1}{\left(\frac{n}{3}\right)}\right)^{\left(\frac{n}{3}\right)}\right]^{\frac{3}{2}} = e^{3/2}$

$\boxed{334:10}$

x	$(1+x)^{1/x}$
10	1.27098
100	1.04723
1000	1.00693
10,000	1.00092
⋮	⋮

guess :

$\lim_{x\to\infty} (1+x)^{1/x} = 1$

$\boxed{334:11}$ $A = P\left(1+\frac{r}{n}\right)^{nt}$, $P = \$1000$, $r = 50\% = 0.5$, $t = 1$ yr. $\rightarrow$ $A = 1000\left(1+\frac{0.5}{n}\right)^{n}$

a.) $n=1 \rightarrow A = \$1500$

b.) $n=2 \rightarrow A = \$1562.50$

c.) $n=12 \rightarrow A = \$1632.09$

d.) $n = 365 \rightarrow A = \$1648.16$

e.) $A = Pe^{rt} = 1000\, e^{(0.5)(1)} = \$1648.72$

$\boxed{334:12}$ $r = 8\% = 0.08$, $n = 1$, $P = \$1000 \rightarrow A = 1000\left(1 + \frac{0.08}{n}\right)^n$

a.) $n = 1 \rightarrow A = \$1080$

b.) $n = 2 \rightarrow A = \$1081.60$

c.) $n = 12 \rightarrow A = \$1082.99$

d.) $n = 365 \rightarrow A = \$1083.28$

e.) $A = Pe^{rt} = 1000\, e^{(0.08)(1)} = \$1083.29$