HW #23

Section 6.3

343:2 $D \ln(1+x^3) = \frac{1}{1+x^3} \cdot 3x^2$

343:3 $D\, x^2 \ln x = x^2 \cdot \frac{1}{x} + 2x \cdot \ln x$

343:5 $D \frac{\ln x}{x} = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$

343:6 $D (\ln x)^3 = 3(\ln x)^2 \cdot \frac{1}{x}$

343:8 $D \sec 5x \cdot \ln 2x$

$= \sec 5x \cdot \frac{1}{2x} \cdot 2 + \sec 5x \cdot \tan 5x \cdot 5 \cdot \ln 2x$

343:10 $D \cos(\ln x) = -\sin(\ln x) \cdot \frac{1}{x}$

343:16 $D \ln(x+\sqrt{x^2+1}) = \frac{1 + \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x}{x + \sqrt{x^2+1}}$

$= \frac{1 + \frac{x}{\sqrt{x^2+1}}}{\frac{x+\sqrt{x^2+1}}{1}} = \frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1}} \cdot \frac{1}{x+\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$

343:22 $D \ln \frac{(2x+1)^{1/2}(3x+2)^{1/3}}{(x^2+1)^5}$

$= D\left[\frac{1}{2}\ln(2x+1) + \frac{1}{3}\ln(3x+2) - 5\ln(x^2+1)\right]$

$= \frac{1}{2} \cdot \frac{2}{2x+1} + \frac{1}{3} \cdot \frac{3}{3x+2} - 5 \cdot \frac{2x}{x^2+1}$

343:24 $D \log_2 [(x^2+1)^3 \sin 3x]$

$= D[3 \log_2(x^2+1) + \log_2(\sin 3x)]$

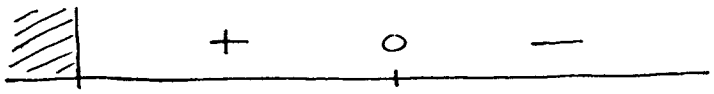
$= 3 \cdot \frac{2x}{x^2+1} \cdot \log_2 e + \frac{\cos 3x \cdot 3}{\sin 3x} \cdot \log_2 e$

343 : 25 $\quad y = \dfrac{\ln x}{x^2}, \quad x > 0$

$$y' = \frac{x^2 \cdot \frac{1}{x} - 2x \ln x}{x^4} = \frac{x - 2x \ln x}{x^4}$$

$$= \frac{x(1 - 2\ln x)}{x^4} = \frac{1 - 2\ln x}{x^3} = 0 \rightarrow$$

$1 - 2\ln x = 0 \rightarrow \ln x = \frac{1}{2} \rightarrow x = e^{1/2}$;



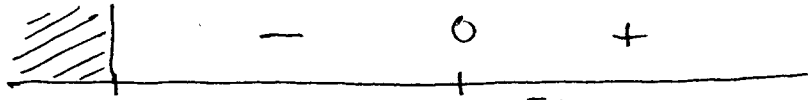
$x = e^{1/2} \approx 1.65$
$y = \frac{1}{2e} \approx 0.18$ } abs. max.

$$y'' = \frac{x^3\left(\frac{-2}{x}\right) - (1 - 2\ln x) \cdot 3x^2}{x^6}$$

$$= \frac{-2x^2 - 3x^2(1 - 2\ln x)}{x^6} = \frac{-x^2\{2 + 3(1 - 2\ln x)\}}{x^6}$$

$$= \frac{-\{5 - 6\ln x\}}{x^4} = 0 \rightarrow 5 - 6\ln x = 0 \rightarrow$$

$\ln x = \frac{5}{6} \rightarrow x = e^{5/6}$;



$x = e^{5/6} \approx 2.3$
$y = \frac{5}{6e^{5/3}} \approx 0.16$ } inf. pt.

Y is $\uparrow$ for $0<X<e^{1/2}$

Y is $\downarrow$ for $X>e^{1/2}$,

Y is $\cup$ for $X>e^{5/6}$,

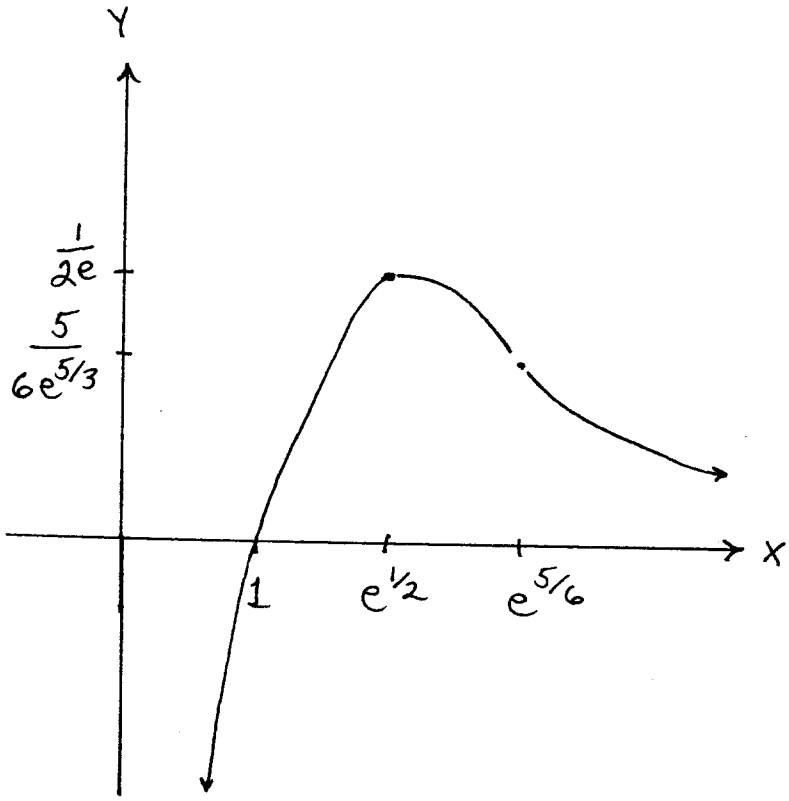
Y is $\cap$ for $0<X<e^{5/6}$,

$$\lim_{X\to 0^+} \frac{\ln X}{X^2} = -\infty,$$

$$\lim_{X\to +\infty} \frac{\ln X}{X^2} = 0,$$

$Y=0$: $\ln X = 0 \to$

$X = 1$.



343:28 $\quad Y = (1+x^2)^{1/2}\cdot(1+\cos 3x)^{5/3} \to$

$$\ln Y = \frac{1}{2}\ln(1+x^2) + \frac{5}{3}\ln(1+\cos 3x) \to$$

$$\frac{1}{Y}Y' = \frac{1}{2}\cdot\frac{2x}{1+x^2} + \frac{5}{3}\cdot\frac{-\sin 3x\cdot 3}{1+\cos 3x} \to$$

$$Y' = (1+x^2)^{1/2}(1+\cos 3x)^{5/3}\cdot\left[\frac{x}{1+x^2} - \frac{5\sin 3x}{1+\cos 3x}\right]$$

343:30 $\quad Y = \dfrac{\cot^3 x}{x^{1/3}\cdot(x^3+2)^{5/2}} \to$

$$\ln Y = 3\ln(\cot x) - \frac{1}{3}\ln x - \frac{5}{2}\ln(x^3+2) \to$$

$$\frac{1}{Y}Y' = 3\cdot\frac{-\csc^2 x}{\cot x} - \frac{1}{3}\cdot\frac{1}{x} - \frac{5}{2}\cdot\frac{3x^2}{x^3+2} \to$$

$$Y' = \frac{\cot^3 x}{x^{1/3}(x^3+2)^{5/2}}\cdot\left[-3\frac{\csc^2 x}{\cot x} - \frac{1}{3x} - \frac{15x^2}{2(x^3+2)}\right]$$

343:36 Let $f(x) = \ln x$ and $x : 1 \to 1+h$, $\Delta x = h$,

$f'(x) = \frac{1}{x}$; $\Delta f = f(1+h) - f(1)$

$= \ln(1+h) - \ln 1 = \ln(1+h)$,

$df = f'(x) \cdot \Delta x = f'(1) \cdot \Delta x = (1) \cdot h = h$, then

by theorem $\Delta f \approx df$, i.e., $\ln(1+h) \approx h$.

Math 21A
Kouba
Worksheet 4

1.) You wish for \$500 in a savings account with no additional deposits to grow to \$1200 in 8 years. If interest is compounded daily, what should the annual interest rate r be ?

2.) A savings account with no additional deposits grew from \$1000 to \$5200. If the annual interest rate was 3.5% compounded yearly, how long was the money in this account ?

3.) An account with interest compounded continuously earned 5.5% annual interest for 3 years. If the final amount in the account was \$12,850 and no additional deposits were made, what was the initial amount ?

4.) An account with interest compounded continuously earned 12% annual interest. If the account grew from \$2000 to \$20,000 and no additional deposits were made, how long was the money in the account ?

5.) A child inherits \$50,000 which is to be deposited in a retirement account. Account A offers an annual interest rate of 5.75% compounded continuously. Account B offers an annual interest rate of 5.8% compounded once per year. Compare the amounts which would be in each account after $t = 5$ years, $t = 50$ years, and $t = 75$ years.

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Worksheet 4

1.) $A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow 1200 = 500\left(1+\frac{r}{365}\right)^{365(8)} \rightarrow$

$\frac{12}{5} = \left(1+\frac{r}{365}\right)^{2920} \rightarrow \left(\frac{12}{5}\right)^{\frac{1}{2920}} = \left(1+\frac{r}{365}\right)^{2920\cdot\frac{1}{2920}} \rightarrow$

$1+\frac{r}{365} = \left(\frac{12}{5}\right)^{\frac{1}{2920}} \rightarrow r = 365\left[\left(\frac{12}{5}\right)^{\frac{1}{2920}} - 1\right]$

$= 0.10945 = 10.945\%$

2.) $A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow 5200 = 1000\left(1+\frac{0.035}{1}\right)^{1\cdot t} \rightarrow$

$5.2 = 1.035^{t} \rightarrow \ln 5.2 = \ln 1.035^{t} \rightarrow$

$\ln 5.2 = t \ln 1.035 \rightarrow t = \frac{\ln 5.2}{\ln 1.035} \approx 47.9$ yrs.

3.) $A = Pe^{rt} \rightarrow 12{,}850 = Pe^{(0.055)(3)} \rightarrow$

$P = \frac{12{,}850}{e^{0.165}} \approx \$10{,}895.43$

4.) $A = Pe^{rt} \rightarrow 20{,}000 = 2000e^{0.12t} \rightarrow 10 = e^{0.12t} \rightarrow$

$\ln 10 = \ln e^{0.12t} \rightarrow \ln 10 = 0.12t \rightarrow t = \frac{\ln 10}{0.12} \approx 19.2$ yrs.

5.) account A: $A = Pe^{rt} = 50{,}000e^{0.0575t}$

account B: $A = P\left(1+\frac{r}{n}\right)^{nt} = 50{,}000\left(1+\frac{0.058}{1}\right)^{1\cdot t}$

	5 yrs.	50 yrs.	75 yrs.
account A	\$66,654.53	\$886,271.21	\$3,731,341.10
account B	\$66,282.42	\$838,018.08	\$3,430,797.50
Difference	\$372.11	\$48,253.13	\$300,543.60