HW # 24

Section 6.5

$\boxed{355:2}$ $D(xe^{-4x}) = x \cdot e^{-4x} \cdot (-4) + e^{-4x} \cdot (1)$

$\boxed{355:4}$ $D \sin 2x \cdot \sin e^{-x}$

$= \sin 2x \cdot \cos e^{-x} \cdot e^{-x} \cdot (-1) + 2\cos 2x \cdot \sin e^{-x}$

$\boxed{355:7}$ $Y = x^{(x^2)} \rightarrow \ln Y = \ln x^{(x^2)} = x^2 \ln x \rightarrow$

$\frac{1}{Y} Y' = x^2 \cdot \frac{1}{x} + 2x \ln x \rightarrow$

$Y' = x^{(x^2)} \cdot (x + 2x \ln x)$.

$\boxed{355:5}$ $D\left(2^{-x^2}\right) = 2^{-x^2} \cdot (-2x) \cdot \ln 2$

$\boxed{355:9}$ $Y = x^{\tan 3x} \rightarrow \ln Y = \ln x^{\tan 3x} = \tan 3x \cdot \ln x \rightarrow$

$\frac{1}{Y} Y' = \tan 3x \cdot \frac{1}{x} + \sec^2 3x \cdot 3 \cdot \ln x \rightarrow$

$Y' = x^{\tan 3x} \left(\frac{1}{x} \tan 3x + 3 \sec^2 3x \cdot \ln x \right)$

$\boxed{355:15}$ $D \ln(x + \sqrt{1+e^{3x}})$

$= \frac{1}{x + \sqrt{1+e^{3x}}} \cdot \left\{ 1 + \frac{1}{2}(1+e^{3x})^{-\frac{1}{2}} \cdot e^{3x} \cdot 3 \right\}$

$\boxed{355:17}$ $D\ e^{ax}\left(\frac{1}{a}x - \frac{1}{a^2}\right)$

$= e^{ax}\left(\frac{1}{a}\right) + e^{ax} \cdot (a) \cdot \left(\frac{1}{a}x - \frac{1}{a^2}\right) = e^{ax} \cdot x$

$\boxed{355:22}$ $f(x) = e^x$, $x: 1 \rightarrow 1.1$, $dx = +0.1$,

$f'(x) = e^x$; then

$\Delta f = f(1.1) - f(1) = e^{1.1} - e^1$ and

$df = f'(1)\, dx = e^1 (0.1) \approx (2.718)(0.1) = 0.2718$;

$\Delta f \approx df \rightarrow$

$e^{1.1} - e \approx 0.2718 \rightarrow e^{1.1} \approx e + 0.2718 \approx 2.99$

$\boxed{355:26}$ $f(x) = \ln x$, $x: 1 \rightarrow 1.1$, $dx = +0.1$, $f'(x) = \frac{1}{x}$,

$\Delta f = f(1.1) - f(1) = \ln 1.1 - \ln 1 = \ln 1.1$ and

$df = f'(1)\, dx = (1)(0.1) = 0.1$; $\Delta f \approx df \rightarrow$

$\ln 1.1 \approx 0.1$

$\boxed{355:30}$ $Y = x^2 e^{-x} \rightarrow$

$Y' = x^2 \cdot e^{-x}(-1) + 2x e^{-x}$

$= x e^{-x}(2-x) = 0$;

−	0	+	0	−	Y'
	$x=0$		$x=2$		

abs. min. $\left\{ \begin{array}{l} x=0 \\ Y=0 \end{array} \right.$ $\left. \begin{array}{l} x=2 \\ Y=\frac{4}{e^2} \approx 0.54 \end{array} \right\}$ rel. max.

$Y'' = 1 \cdot e^{-x}(2-x) + x \cdot e^{-x}(-1)(2-x) + x e^{-x} \cdot (-1)$

$= e^{-x}[2 - x - 2x + x^2 - x] = e^{-x}(x^2 - 4x + 2) = 0 \rightarrow$

$$x = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2} \ ;$$

+	0	−	0	+	Y''
	$x = 2-\sqrt{2} \approx 0.59$		$x = 2+\sqrt{2} \approx 3.41$		
	$\underbrace{Y \approx 0.19}$		$\underbrace{Y \approx 0.38}$		

← inf. pts. →

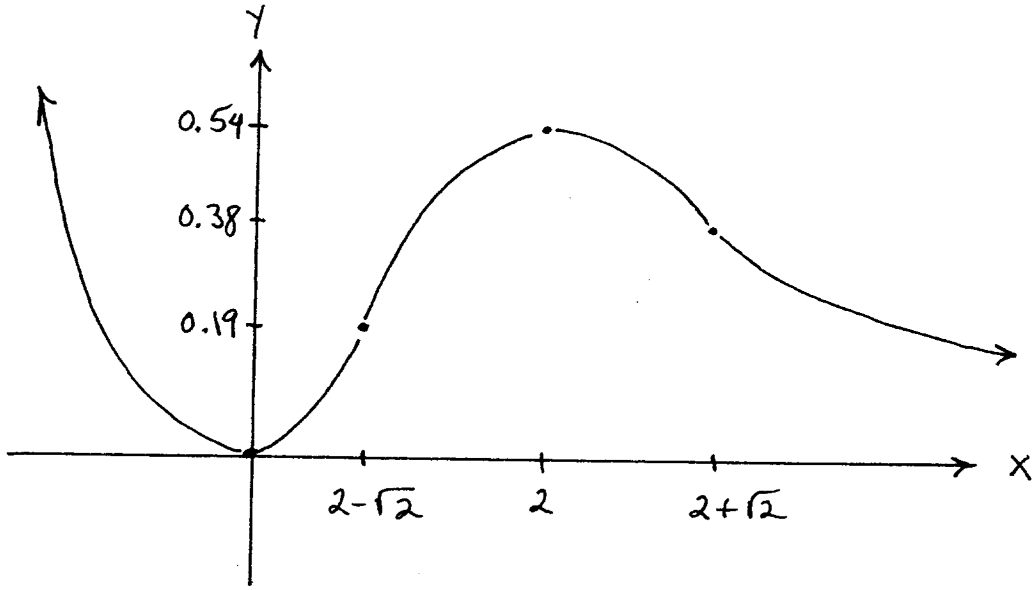
Y is ↑ for $0 < x < 2$,

Y is ↓ for $x < 0$, $x > 2$,

Y is ∪ for $x < 2-\sqrt{2}$, $x > 2+\sqrt{2}$,

Y is ∩ for $2-\sqrt{2} < x < 2+\sqrt{2}$,

$\lim_{x \to +\infty} x^2 e^{-x} = 0$, $\lim_{x \to -\infty} x^2 e^{-x} = +\infty$;



$\boxed{355:40}$ $f(x) = 5 + (x - x^2)e^x$

a.) $f(0) = 5 = f(1)$

b.) f is cont. on $[0,1]$ since it is sum and product of continuous fcns., and diff. on $(0,1)$ with

$f'(x) = (x-x^2)e^x + (1-2x)e^x = e^x(-x^2 - x + 1)$; by

MVT ~~there~~ is a c, $0 < c < 1$, satisfying

$$f'(c) = \frac{f(1) - f(0)}{1 - 0} = \frac{5-5}{1} = 0 \rightarrow$$

$$e^c(-c^2 - c + 1) = 0 \rightarrow c = \frac{1 \pm \sqrt{1+4}}{-2} = \frac{1-\sqrt{5}}{-2} \approx 0.618$$

$\boxed{355:42a}$ $3x + \sin x - e^x = 0$, let $m=0$ and $f(x) = 3x + \sin x - e^x$, which is continuous for all x-values since it is the sum and difference of continuous functions.

Since $f(0) = 0 + \sin 0 - e^0 = -1$ and $f(1) = 3 + \sin 1 - e > 0$ and $m=0$ is between $f(0)$ and $f(1)$, it follows from the IMVT that there is at least one number c, $0<c<1$, so that $f(c)=0$, i.e., the original equation is solvable.