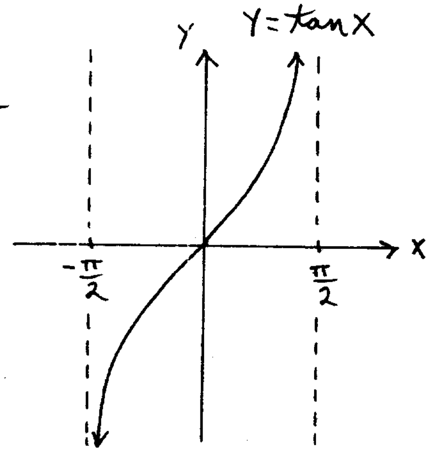
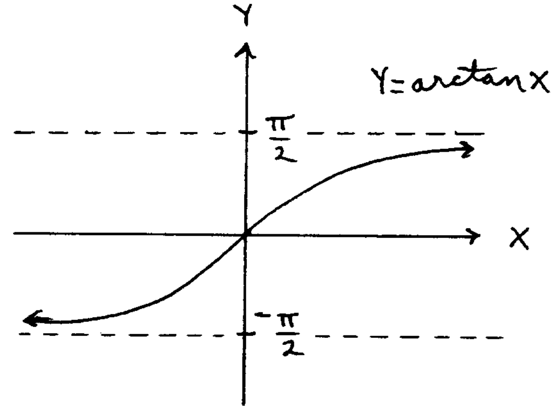
Section 6.6

$\boxed{365:4}$





a.) $\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$ b.) $\lim_{x\to-\infty} \arctan x = -\frac{\pi}{2}$

$\boxed{365:13}$ $\sin(\tan^{-1} 1) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\boxed{365:14}$ $\tan(\sec^{-1} 2) = \tan\left(\cos^{-1}\frac{1}{2}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\boxed{365:15}$ $\tan\left(\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right) = \tan\left(-\frac{\pi}{4}\right) = -1$

$\boxed{365:16}$ $\tan\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\boxed{365:17}$ $\sin(\sin^{-1} 0.3) = 0.3$

$\boxed{365:18}$ $\sin(\tan^{-1} 0) = \sin(0) = 0$

$\boxed{365:22}$ $D \tan^{-1}(3x) = \frac{1}{1+(3x)^2} \cdot 3$

$\boxed{365:26}$ $D\ \frac{-1}{3}\sin^{-1}\left(\frac{3}{x}\right) = \frac{-1}{3} \cdot \frac{1}{\sqrt{1-\left(\frac{3}{x}\right)^2}} \cdot \frac{-3}{x^2}$

$\boxed{365:27}$ $D\ x^2 \sec^{-1}\sqrt{x} = x^2 \cdot \frac{1}{\sqrt{x}\sqrt{(\sqrt{x})^2-1}} \cdot \frac{1}{2\sqrt{x}} + 2x \cdot \sec^{-1}\sqrt{x}$

$\boxed{365:36}$ $D \ln(\sin^{-1} 5x)^2 = \frac{1}{(\sin^{-1} 5x)^2} \cdot 2(\sin^{-1} 5x) \cdot \frac{1}{\sqrt{1-(5x)^2}} \cdot 5$

$\boxed{365:38}$ $D\ 10^{\sec^{-1} 2x} = 10^{\sec^{-1} 2x} \cdot \ln 10 \cdot \frac{1}{|2x|\sqrt{(2x)^2-1}} \cdot 2$

$\boxed{365:40}$ $D\ 2^x \cdot \log_3 x \cdot \sec 3x = (2^x \ln 2) \log_3 x \cdot \sec 3x$
$+ 2^x \cdot \left(\frac{1}{x} \log_3 e\right) \cdot \sec 3x + 2^x \log_3 x \cdot (\sec 3x \tan 3x \cdot 3)$

$\boxed{365:42}$ $D\left(\sin^{-1}\sqrt{x-1}\right)^4 = 4\left(\sin^{-1}\sqrt{x-1}\right)^3 \cdot \frac{1}{\sqrt{1-(\sqrt{x-1})^2}} \cdot \frac{1}{2\sqrt{x-1}}$

$\boxed{365:47}$ $D\left(\sqrt{1+x}\cdot\sqrt{2-x} - 3\sin^{-1}\sqrt{\frac{2-x}{3}}\right)$

$$= \sqrt{1+x}\cdot\frac{1}{2\sqrt{2-x}}(-1) + \frac{1}{2\sqrt{1+x}}\cdot\sqrt{2-x} - 3\cdot\frac{1}{\sqrt{1-\left(\sqrt{\frac{2-x}{3}}\right)^2}}\cdot\frac{1}{2\sqrt{\frac{2-x}{3}}}\cdot\frac{-1}{3}$$

$$= \frac{-\sqrt{1+x}}{2\sqrt{2-x}} + \frac{\sqrt{2-x}}{2\sqrt{1+x}} + \frac{1}{\sqrt{1-\frac{2-x}{3}}}\cdot\frac{1}{2\sqrt{\frac{2-x}{3}}}$$

$$= \frac{-(1+x)+(2-x)}{2\sqrt{2-x}\sqrt{1+x}} + \frac{1}{\sqrt{\frac{1+x}{3}}}\cdot\frac{1}{2\sqrt{\frac{2-x}{3}}}$$

$$= \frac{(-1-x+2-x)+3}{2\sqrt{2-x}\sqrt{1+x}} = \frac{4-2x}{2\sqrt{2-x}\sqrt{1+x}} = \frac{2-x}{\sqrt{2-x}\sqrt{1+x}} = \frac{\sqrt{2-x}}{\sqrt{1+x}}$$

$\boxed{365:50}$ $D\left(x\cdot\tan^{-1}5x - \frac{1}{10}\cdot\ln(1+25x^2)\right)$

$$= x\cdot\frac{1}{1+(5x)^2}\cdot 5 + \tan^{-1}5x - \frac{1}{10}\cdot\frac{50x}{1+25x^2} = \tan^{-1}5x$$

$\boxed{365:54}$ $f(x) = \tan^{-1}x$, $x: 1 \rightarrow 1.1$, $\Delta x = 0.1$,

$f'(x) = \frac{1}{1+x^2}$; $\Delta f = f(1.1) - f(1) = \tan^{-1}(1.1) - \tan^{-1}(1)$
$= \tan^{-1}(1.1) - \frac{\pi}{4}$;

$df = f'(1)\cdot\Delta x = \left(\frac{1}{2}\right)\cdot(0.1) = 0.05$, and by theorem,

$\Delta f \approx df \longrightarrow \tan^{-1}(1.1) - \frac{\pi}{4} \approx 0.05 \longrightarrow$

$$\tan^{-1}(1.1) \approx \frac{\pi}{4} + 0.05 \approx 0.835$$

$\boxed{365:55}$ $f(x)=\sin^{-1}x$, $x: 0.50 \to 0.47$, $\Delta x = -0.03$,

$$f'(x)=\frac{1}{\sqrt{1-x^2}}\,; \quad \Delta f = f(0.47)-f(0.50)$$
$$= \sin^{-1}(0.47)-\sin^{-1}(0.50)$$
$$= \sin^{-1}(0.47)-\frac{\pi}{6}$$

$df = f'(0.5)\cdot\Delta x = \frac{2}{\sqrt{3}}(-0.03)$, by theorem

$\Delta f \approx df \longrightarrow \sin^{-1}(0.47)-\frac{\pi}{6} \approx \frac{2}{\sqrt{3}}(-0.03) \longrightarrow$

$$\sin^{-1}(0.47) \approx \frac{\pi}{6}-\frac{0.06}{\sqrt{3}} \approx 0.489$$

$\boxed{365:57}$ Recall: $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \tan B}$.

Show $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\frac{\pi}{4}$. First note that $\theta = \tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}$ satisfies $\frac{\pi}{2}<\theta<-\frac{\pi}{2}$.

Then
$$\tan\theta = \tan\left(\tan^{-1}\tfrac{1}{2}+\tan^{-1}\tfrac{1}{3}\right)$$
$$= \frac{\tan(\tan^{-1}\frac{1}{2})+\tan(\tan^{-1}\frac{1}{3})}{1-\tan(\tan^{-1}\frac{1}{2})\cdot\tan(\tan^{-1}\frac{1}{3})}$$
$$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1.$$
Thus $\theta = \frac{\pi}{4}$.

$\boxed{365:65}$ a.) $\sin(\arcsin x) = x$ for x in $[-1,1]$

b.) $\arcsin(\sin x) = x$ for x in $[-\frac{\pi}{2}, \frac{\pi}{2}]$