

Section 6.8

$$\boxed{384:4} \quad \lim_{x \rightarrow 0} \frac{\sin x^2}{(\sin x)^2} \stackrel{''0/0''}{=} \lim_{x \rightarrow 0} \frac{2x \cdot \cos x^2}{2 \sin x \cos x}$$

$$\stackrel{''0/0''}{=} \lim_{x \rightarrow 0} \frac{2x \cdot (-\sin x^2) \cdot 2x + 2 \cdot \cos x^2}{-2 \sin^2 x + 2 \cos^2 x} = \frac{2}{2} = 1$$

$$\boxed{384:11} \quad \lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x} \stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow \infty} \frac{2 \ln x \cdot \frac{1}{x}}{1}$$

$$= \lim_{x \rightarrow \infty} \frac{2 \ln x}{x} \stackrel{''\infty/\infty''}{=} \lim_{x \rightarrow \infty} \frac{2 \cdot \frac{1}{x}}{1} = \frac{2 \cdot 0}{1} = 0$$

$$\boxed{384:12} \quad \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{e^{2x} - 1} \stackrel{''0/0''}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}}}{2e^{2x}} = \frac{1}{2}$$

$$\boxed{384:13} \quad \lim_{x \rightarrow 0} (1-2x)^{\frac{1}{x}} = "1 \pm \infty"$$

$$= \lim_{x \rightarrow 0} e^{\ln(1-2x)^{\frac{1}{x}}} = \lim_{x \rightarrow 0} e^{\frac{1}{x} \ln(1-2x)}$$

$$= e^{\lim_{x \rightarrow 0} \frac{\ln(1-2x)}{x} \stackrel{''0/0''}{=} \lim_{x \rightarrow 0} \frac{-2}{1-2x} = e^{-2}}$$

$$\boxed{384:14} \quad \lim_{x \rightarrow 0} (1 + \sin 2x)^{\csc x} = "1 \pm \infty"$$

$$= \lim_{x \rightarrow 0} e^{\ln(1 + \sin 2x)^{\csc x}} = \lim_{x \rightarrow 0} e^{\csc x \cdot \ln(1 + \sin 2x)}$$

$$= \lim_{x \rightarrow 0} e^{\frac{\ln(1 + \sin 2x)}{\sin x}} = e^{\lim_{x \rightarrow 0} \frac{\ln(1 + \sin 2x)}{\sin x}}$$

$$\stackrel{''0/0''}{=} e^{\lim_{x \rightarrow 0} \frac{2 \cos 2x}{1 + \sin 2x} \cdot \frac{1}{\cos x}} = e^2$$

$$\boxed{384:15} \quad \lim_{x \rightarrow 0^+} (\sin x)^{e^x - 1} = "0^0"$$

$$= \lim_{x \rightarrow 0^+} e^{\ln(\sin x)^{e^x - 1}} = e^{\lim_{x \rightarrow 0^+} (e^x - 1) \cdot \ln(\sin x)}$$

$$= e^{\lim_{x \rightarrow 0^+} \frac{\ln(\sin x)}{\frac{1}{e^x - 1}} \stackrel{"\infty/\infty"}{=} \lim_{x \rightarrow 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{-e^x}{(e^x - 1)^2}}$$

$$= e^{\lim_{x \rightarrow 0^+} \frac{(e^x - 1)^2 \cos x}{-e^x \sin x} = \frac{0}{0}}$$

$$= e^{\lim_{x \rightarrow 0^+} \frac{-\sin x \cdot (e^x - 1)^2 + 2e^x(e^x - 1) \cdot \cos x}{-[e^x \cdot \cos x + e^x \cdot \sin x]}}$$

$$= e^{\frac{0+0}{-(1+0)}} = e^0 = 1.$$

$$\boxed{384:16} \quad \lim_{x \rightarrow 0^+} x^2 \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x^2}}$$

$$\stackrel{"\infty/\infty"}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x \rightarrow 0^+} \frac{x^2}{-2} = 0.$$

$$\boxed{384:17} \quad \lim_{x \rightarrow 0^+} (\tan x)^{\tan 2x} = "0^0"$$

$$= \lim_{x \rightarrow 0^+} e^{\ln(\tan x)^{\tan 2x}} = \lim_{x \rightarrow 0^+} e^{\tan 2x \cdot \ln(\tan x)}$$

$$= e^{\lim_{x \rightarrow 0^+} \frac{\ln(\tan x)}{\cot 2x} \stackrel{"\infty/\infty"}{=} \lim_{x \rightarrow 0^+} \frac{\frac{\sec^2 x}{\tan x}}{-2 \csc^2 2x}}$$

$$= e^{\lim_{x \rightarrow 0^+} \frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x} \cdot \frac{\sin^2 2x}{-2}}$$

$$= e^{\lim_{x \rightarrow 0^+} \frac{1}{\cos x \sin x} \cdot \frac{(2 \sin x \cos x)^2}{-2}}$$

$$= e^{\lim_{x \rightarrow 0^+} -2 \cdot \sin x \cdot \cos x} = e^{-2(0)(1)} = e^0 = 1.$$

$$\boxed{384:19} \quad \lim_{x \rightarrow \infty} \frac{2^x}{3^x} = \lim_{x \rightarrow \infty} \left(\frac{2}{3}\right)^x = 0.$$

$$\boxed{384:20} \quad \lim_{x \rightarrow \infty} \frac{2^x + x}{3^x} = \lim_{x \rightarrow \infty} \left(\frac{2}{3}\right)^x + \lim_{x \rightarrow \infty} \frac{x}{3^x}$$

" $\frac{\infty}{\infty}$ "

$$= 0 + \lim_{x \rightarrow \infty} \frac{1}{3^x \cdot \ln 3} = \frac{1}{\infty} = 0.$$

$$\boxed{384:21} \quad \lim_{x \rightarrow \infty} \frac{\log_2 x}{\log_3 x} \stackrel{\frac{\infty}{\infty}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x} \cdot \log_2 e}{\frac{1}{x} \cdot \log_3 e} = \frac{\log_2 e}{\log_3 e}$$

$$\boxed{384:22} \quad \lim_{x \rightarrow 1} \frac{\log_2 x}{\log_3 x} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 1} \frac{\frac{1}{x} \cdot \log_2 e}{\frac{1}{x} \cdot \log_3 e} = \frac{\log_2 e}{\log_3 e}.$$

$$\boxed{384:23} \quad \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{1}{\sin x}\right) = \lim_{x \rightarrow \infty} \frac{1}{x} - \lim_{x \rightarrow \infty} \frac{1}{\sin x}$$

$$= 0 - \lim_{x \rightarrow \infty} \frac{1}{\sin x} \quad \text{does not exist.}$$

$$\boxed{384:24} \quad \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+3} - \sqrt{x^2+4x})(\sqrt{x^2+3} + \sqrt{x^2+4x})}{(\sqrt{x^2+3} + \sqrt{x^2+4x})}$$

$$= \lim_{x \rightarrow \infty} \frac{(x^2+3) - (x^2+4x)}{\sqrt{x^2+3} + \sqrt{x^2+4x}} = \lim_{x \rightarrow \infty} \frac{\frac{3}{x} - 4}{\sqrt{1+\frac{3}{x^2}} + \sqrt{1+\frac{4}{x}}} = \frac{0-4}{2} = -2$$

$$\boxed{384:25} \quad \lim_{x \rightarrow \infty} \frac{x^2 + 3 \cos 5x}{x^2 - 2 \sin 4x}$$

$$= \lim_{x \rightarrow \infty} \frac{1 + \frac{3 \cos 5x}{x^2}}{1 - \frac{2 \sin 4x}{x^2}} = \frac{1+0}{1-0} = 1$$

$$\boxed{384:26} \quad \lim_{x \rightarrow \infty} \frac{e^x - \frac{1}{x}}{e^x + \frac{1}{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x e^x}}{1 + \frac{1}{x e^x}} = \frac{1-0}{1+0} = 1$$

$$\boxed{384:29} \quad \lim_{x \rightarrow \infty} \frac{\sin x}{4 + \sin x} = \lim_{x \rightarrow \infty} \frac{1}{1 + 4 \csc x}$$

does not exist since $\lim_{x \rightarrow \infty} 4 \csc x$ does not exist.

$$\boxed{384:38} \quad \lim_{x \rightarrow 0} \frac{x e^x \cos^2 6x}{e^{2x} - 1}$$

$$\begin{aligned} & \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{e^x \cos^2 6x + x e^x \cos^2 6x - 12 x e^x \cos 6x \cdot \sin 6x}{2 e^{2x}} \end{aligned}$$

$$= \frac{1+0-0}{2} = \frac{1}{2}$$

$$\boxed{384:45} \quad \lim_{x \rightarrow 0} \left(\frac{1}{1 - \cos x} - \frac{2}{x^2} \right) = \text{"} \infty - \infty \text{"}$$

$$= \lim_{x \rightarrow 0} \frac{x^2 - 2(1 - \cos x)}{x^2(1 - \cos x)}$$

$$\begin{aligned} & \text{"0/0"} \\ & = \lim_{x \rightarrow 0} \frac{2x - 2 \sin x}{x^2 \sin x + 2x(1 - \cos x)} \end{aligned}$$

$$\begin{aligned} & \text{"0/0"} \\ & = \lim_{x \rightarrow 0} \frac{2 - 2 \cos x}{x^2 \cos x + 2x \sin x + 2x \sin x + 2(1 - \cos x)} \end{aligned}$$

$$= \lim_{x \rightarrow 0} \frac{2 - 2 \cos x}{x^2 \cos x + 4x \sin x - 2 \cos x + 2}$$

$$\begin{aligned} & \text{"0/0"} \\ & = \lim_{x \rightarrow 0} \frac{2 \sin x}{-x^2 \sin x + 2x \cos x + 4x \cos x + 4 \sin x + 2 \sin x} \end{aligned}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin x}{-x^2 \sin x + 6x \cos x + 6 \sin x}$$

$$\begin{aligned} & \text{"0/0"} \\ & = \lim_{x \rightarrow 0} \frac{2 \cos x}{-x^2 \cos x - 2x \sin x - 6x \sin x + 6 \cos x + 6 \cos x} \end{aligned}$$

$$= \frac{2}{0 - 0 - 0 + 6 + 6} = \frac{1}{6}$$

$$\boxed{384:46} \quad \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{\tan^{-1} 2x} \stackrel{\text{"0/0"}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-x^2}}}{\frac{2}{1+4x^2}} = \frac{1}{2}$$

$$\boxed{384:53} \quad \lim_{x \rightarrow 0} \left(\frac{1+2^x}{2} \right)^{\frac{1}{x}} = \text{"1} \pm \infty \text{"}$$

$$= \lim_{x \rightarrow 0} e^{\frac{\ln \left(\frac{1}{2} + 2^{x-1} \right)}{x}} = \lim_{x \rightarrow 0} e^{\frac{\ln \left(\frac{1}{2} + 2^{x-1} \right)}{x}}$$

$$= e^{\lim_{x \rightarrow 0} \frac{\ln \left(\frac{1}{2} + 2^{x-1} \right)}{x}} = e^{\lim_{x \rightarrow 0} \frac{2^{x-1} \ln 2}{\frac{1}{2} + 2^{x-1}}}$$

$$= e^{\frac{1}{2} \ln 2} = e^{\ln \sqrt{2}} = \sqrt{2}$$

384:56 $f(x) = (1+x)^{\frac{1}{x}}$ for $x \neq 0$;
for $x > -1$;

$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$; and

$\lim_{x \rightarrow \infty} \ln(1+x)^{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\ln(1+x)}{x}$

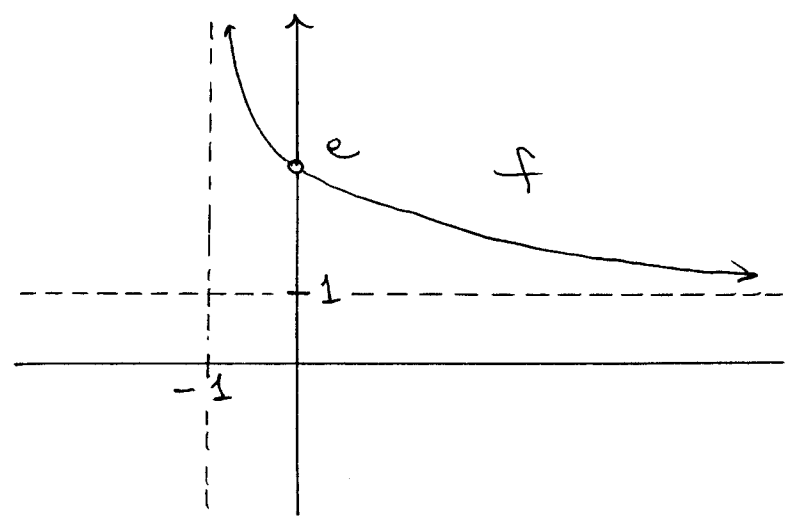
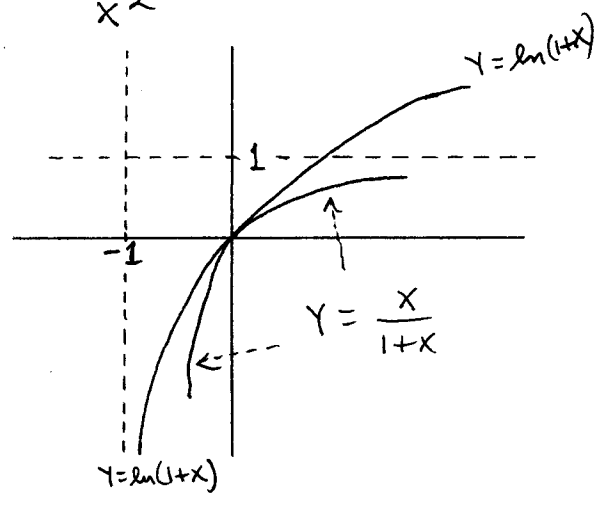
" $\frac{\infty}{\infty}$ " $\lim_{x \rightarrow \infty} \frac{1}{1+x} = 0$ so $\lim_{x \rightarrow \infty} (1+x)^{\frac{1}{x}} = 1$;

$\ln Y = \frac{\ln(1+x)}{x} \xrightarrow{D} \frac{1}{Y} Y' = \frac{x \cdot \frac{1}{1+x} - \ln(1+x)}{x^2} \rightarrow$

$Y' = (1+x)^{\frac{1}{x}} \cdot \frac{\frac{x}{1+x} - \ln(1+x)}{x^2}$

< 0 for $x > -1, x \neq 0$;

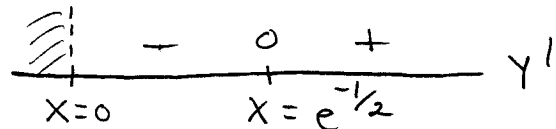
$\lim_{x \rightarrow -1^+} (1+x)^{\frac{1}{x}} = (0^+)^{-1} = +\infty$



384:58 $y = x^2 \ln x, x > 0$

$$y' = x^2 \left(\frac{1}{x}\right) + 2x \ln x = x + 2x \ln x$$

$$= x(1 + 2 \ln x) = 0$$

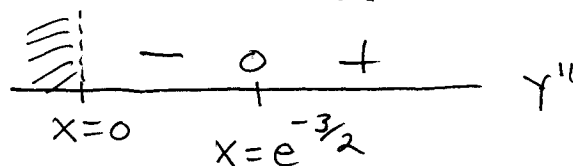


$$y'' = 1 + 2x \left(\frac{1}{x}\right) + 2 \ln x$$

$$= 3 + 2 \ln x = 0$$

$y = \frac{-1}{2} e^{-1}$
abs. min

$$y = 0 \rightarrow x = 1$$



y is \uparrow for $x > e^{-1/2}$
 y is \downarrow for $0 < x < e^{-1/2}$,
 y is \cup for $x > e^{-3/2}$,
 y is \cap for $0 < x < e^{-3/2}$

$y = \frac{-3}{2} e^{-3}$
infl. pt.

$$\lim_{x \rightarrow 0^+} x^2 \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x^2}} \stackrel{\text{"-\infty/\infty"}}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$$

$$= \lim_{x \rightarrow 0^+} \frac{-1}{2} x^2 = 0$$

