# HW #3

## Section 2.4

$\boxed{44:6}$ $\lim_{x\to 5} (x^2-x)(2x-7) = (20)(3) = 60$

$\boxed{44:9}$ $\lim_{x\to +\infty} (4x^2-x+3) = \lim_{x\to+\infty} \left(x(4x-1)+3\right) = +\infty$

$\boxed{44:11}$ $\lim_{x\to +\infty} (x^5-100x^4) = \lim_{x\to+\infty} x^4(x-100) = +\infty$

$\boxed{44:12}$ $\lim_{x\to +\infty} (-4x^5+35x^2) = \lim_{x\to+\infty} x^2(35-4x^3) = -\infty$

$\boxed{44:13}$ $\lim_{x\to -\infty} (6x^5+21x^3) = \lim_{x\to-\infty} 3x^3(2x^2+7) = -\infty$

$\boxed{44:14}$ $\lim_{x\to -\infty} (19x^6+5x) = \lim_{x\to-\infty} x(19x^5+5) = (-\infty)(-\infty) = +\infty$

$\boxed{44:18}$ $\lim_{x\to +\infty} \dfrac{100x^9+22}{x^{10}+21} = \lim_{x\to+\infty} \dfrac{\frac{100}{x}+\frac{22}{x^{10}}}{1+\frac{21}{x^{10}}} = \dfrac{0+0}{1+0} = 0$

$\boxed{44:20}$ $\lim_{x\to +\infty} \dfrac{6x^3-x^2+5}{3x^3-100x+1} = \lim_{x\to+\infty} \dfrac{6-\frac{1}{x}+5/x^3}{3-100/x^2+1/x^3}$

$= \dfrac{6-0+0}{3-0+0} = 2$

$\boxed{44:22}$ $\lim_{x\to -\infty} \dfrac{5x^3+2x}{x^2+x+7} = \lim_{x\to-\infty} \dfrac{5x+\frac{2}{x}}{1+\frac{1}{x}+\frac{7}{x^2}}$

$= \dfrac{-\infty+0}{1+0+0} = -\infty$

$\boxed{44:23}$ $\lim_{x\to 0^+} \dfrac{1}{x^3} = \text{"}\dfrac{1}{0^+}\text{"} = +\infty$

$\boxed{44:24}$ $\lim_{x\to 0^-} \dfrac{1}{x^3} = \text{"}\dfrac{1}{0^-}\text{"} = -\infty$

RECALL: $\sqrt{z^2} = |z| = \begin{cases} z & \text{if } z \geq 0 \\ -z & \text{if } z < 0 \end{cases}$.

$\boxed{44:26}$ $\lim\limits_{x \to 0^-} \dfrac{1}{x^4} = "\dfrac{1}{0^+}" = +\infty$

$\boxed{44:29}$ $\lim\limits_{x \to +\infty} \dfrac{\sqrt{4x^2+2x+1}}{3x} = \lim\limits_{x \to +\infty} \dfrac{\sqrt{4x^2+2x+1}}{\sqrt{(3x)^2}}$

$= \lim\limits_{x \to +\infty} \sqrt{\dfrac{4x^2+2x+1}{9x^2}} = \lim\limits_{x \to +\infty} \sqrt{\dfrac{4}{9} + \dfrac{2}{9x} + \dfrac{1}{9x^2}} = \sqrt{\dfrac{4}{9} + 0 + 0} = \dfrac{2}{3}$

$\boxed{44:30}$ $\lim\limits_{x \to -\infty} \dfrac{\sqrt{9x^2+x+3}}{6x} = \lim\limits_{x \to -\infty} \dfrac{\sqrt{9x^2+x+3}}{-\sqrt{(6x)^2}}$

$= \lim\limits_{x \to -\infty} -\sqrt{\dfrac{9x^2+x+3}{36x^2}} = \lim\limits_{x \to -\infty} -\sqrt{\dfrac{1}{4} + \dfrac{1}{36x} + \dfrac{1}{12x^2}} = -\sqrt{\dfrac{1}{4} + 0 + 0} = \dfrac{-1}{2}$

$\boxed{44:31}$ $\lim\limits_{x \to +\infty} \dfrac{\sqrt{4x^2+x}}{\sqrt{9x^2-3x}} = \lim\limits_{x \to +\infty} \sqrt{\dfrac{4x^2+x}{9x^2-3x} \cdot \dfrac{1/x^2}{1/x^2}}$

$= \lim\limits_{x \to +\infty} \sqrt{\dfrac{4 + 1/x}{9 - 3/x}} = \sqrt{\dfrac{4}{9}} = \dfrac{2}{3}$

$\boxed{44:32}$ $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+3x+1}}{\sqrt{16x^2+x+2}} = \lim\limits_{x \to -\infty} \sqrt{\dfrac{x^2+3x+1}{16x^2+x+2} \cdot \dfrac{1/x^2}{1/x^2}}$

$= \lim\limits_{x \to -\infty} \sqrt{\dfrac{1 + 3/x + 1/x^2}{16 + \frac{1}{x} + 2/x^2}} = \sqrt{\dfrac{1}{16}} = \dfrac{1}{4}$

$\boxed{44:38}$ $\lim\limits_{x \to +\infty} \left( \dfrac{3x^2+2x}{x+5} - 3x \right) = \lim\limits_{x \to +\infty} \dfrac{\cancel{3x^2}+2x-\cancel{3x^2}-15x}{x+5}$

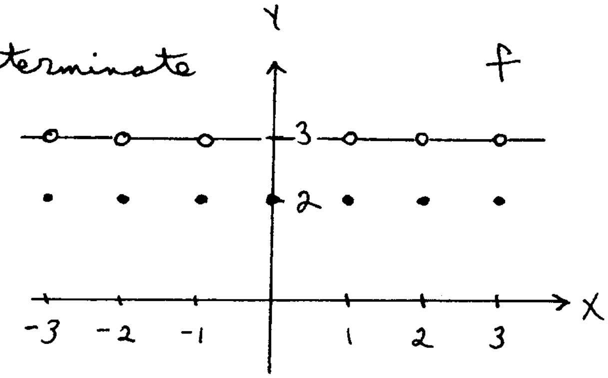
$= \lim\limits_{x \to +\infty} \dfrac{-13x}{x+5} = \lim\limits_{x \to +\infty} \dfrac{-13}{1 + 5/x} = -13$.

$\boxed{44:42}$ a.) $\lim\limits_{x \to +\infty} (f(x) + g(x)) = "+\infty + \infty" = +\infty$

b.) $\lim\limits_{x \to +\infty} (f(x) - g(x)) = "\infty - \infty"$, indeterminate c.) $\lim\limits_{x \to +\infty} f(x)g(x) = "\infty \cdot \infty" = +\infty$

d.) $\lim\limits_{x \to +\infty} \dfrac{g(x)}{f(x)} = "\dfrac{\infty}{\infty}"$, indeterminate

$\boxed{44:45}$ a.) $f(x) = \begin{cases} 2, & x \text{ integer} \\ 3, & x \text{ not integer} \end{cases}$




c.) $\lim\limits_{x \to 2} f(x) = 3$

b.) $\lim\limits_{x \to +\infty} f(x)$ does not exist

$\boxed{44:46}$ $\lim\limits_{x \to +\infty} \left(\sqrt{x^2+100} - x\right) = "\infty - \infty"$

$$= \lim_{x \to +\infty} \frac{\left(\sqrt{x^2+100} - x\right)\left(\sqrt{x^2+100} + x\right)}{\left(\sqrt{x^2+100} + x\right)} = \lim_{x \to +\infty} \frac{\cancel{x^2} + 100 - \cancel{x^2}}{\sqrt{x^2+100} + x}$$

$$= "\frac{100}{\infty + \infty}" = "\frac{100}{\infty}" = 0 .$$

$\boxed{44:47}$ $\lim\limits_{x \to +\infty} \left(\sqrt{x^2+20x} - x\right) = "\infty - \infty"$

$$= \lim_{x \to +\infty} \frac{\left(\sqrt{x^2+20x} - x\right)\left(\sqrt{x^2+20x} + x\right)}{\left(\sqrt{x^2+20x} + x\right)} = \lim_{x \to +\infty} \frac{\cancel{x^2} + 20x - \cancel{x^2}}{\sqrt{x^2+20x} + x}$$

$$= \lim_{x \to +\infty} \frac{20}{\frac{1}{x}\sqrt{x^2+20x} + 1} = \lim_{x \to +\infty} \frac{20}{\sqrt{\frac{x^2+20x}{x^2}} + 1}$$

$$= \lim_{x \to +\infty} \frac{20}{\sqrt{1 + \frac{20}{x}} + 1} = \frac{20}{1+1} = 10$$

## Section 2.7

$\boxed{74:5}$ $\lim_{x\to 0} \frac{\sin 3x}{5x} = \lim_{x\to 0} \frac{3}{5}\cdot\frac{\sin(3x)}{(3x)} = \frac{3}{5}\cdot 1 = \frac{3}{5}$

$\boxed{74:6}$ $\lim_{x\to 0} \frac{2x}{\sin 3x} = \lim_{x\to 0} \frac{2}{3}\cdot\frac{3x}{\sin 3x}$

$= \lim_{x\to 0} \frac{2}{3}\cdot\frac{1}{\frac{\sin(3x)}{(3x)}} = \frac{2}{3}\cdot\frac{1}{1} = \frac{2}{3}$

$\boxed{74:7}$ $\lim_{\theta\to 0} \frac{\sin^2\theta}{\theta} = \lim_{\theta\to 0} \theta\cdot\frac{\sin^2\theta}{\theta^2}$

$= \lim_{\theta\to 0} \theta\cdot\left(\frac{\sin\theta}{\theta}\right)^2 = 0\cdot(1)^2 = 0$

$\boxed{74:8}$ $\lim_{h\to 0} \frac{\sin(h^2)}{(h^2)} = 1$

$\boxed{74:9}$ $\lim_{\theta\to 0} \frac{\tan^2\theta}{\theta} = \lim_{\theta\to 0} \frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{1}{\theta}$

$= \lim_{\theta\to 0} \frac{\sin\theta}{\theta}\cdot\frac{\sin\theta}{\cos^2\theta} = 1\cdot\frac{0}{(1)^2} = 0$

$\boxed{74:10}$ $\lim_{\theta\to 0} \theta\cot\theta = \lim_{\theta\to 0} \theta\cdot\frac{\cos\theta}{\sin\theta}$

$= \lim_{\theta\to 0} \frac{\theta}{\sin\theta}\cdot\cos\theta = \lim_{\theta\to 0} \frac{1}{\frac{\sin\theta}{\theta}}\cdot\cos\theta = \frac{1}{1}\cdot 1 = 1$

$\boxed{74:18}$

| $\theta$ | $\frac{\theta-\sin\theta}{\theta^3}$ |
|---|---|
| 0.001 | 0.16671 |
| 0.0001 | 0.171 |
| −0.0001 | 0.171 |

It can be shown that $\lim_{\theta\to 0} \frac{\theta-\sin\theta}{\theta^3} = \frac{1}{6}$.

$\boxed{74:21}$ $f(x) = \dfrac{\sin x}{x}$  a.) Domain : $x \neq 0$

b.) $f(-x) = \dfrac{\sin(-x)}{(-x)} = \dfrac{-\sin x}{-x} = \dfrac{\sin x}{x} = f(x)$,

so f is an even function.

c.) $\dfrac{-1}{x} \leq \dfrac{\sin x}{x} \leq \dfrac{1}{x}$ for $x > 0$ so

$\lim\limits_{x \to \infty} \left(\dfrac{-1}{x}\right) \leq \lim\limits_{x \to \infty} \left(\dfrac{\sin x}{x}\right) \leq \lim\limits_{x \to \infty} \left(\dfrac{1}{x}\right) \rightarrow$

$0 \leq \lim\limits_{x \to \infty} \dfrac{\sin x}{x} \leq 0$ so

$\lim\limits_{x \to \infty} \dfrac{\sin x}{x} = 0$.

d.) $f(x) = 0 \rightarrow \dfrac{\sin x}{x} = 0 \rightarrow \sin x = 0 \rightarrow$

$x = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$ .

$\boxed{74:22}$ $g(x) = \dfrac{1 - \cos x}{x}$  a.) Domain : $x \neq 0$

b.) $g(-x) = \dfrac{1 - \cos(-x)}{(-x)} = \dfrac{1 - \cos x}{-x} = -\left(\dfrac{1 - \cos x}{x}\right) = -g(x)$,

so g is an odd function

c.) $\dfrac{0}{x} \leq \dfrac{1 - \cos x}{x} \leq \dfrac{2}{x}$ for $x > 0$ so

$\lim\limits_{x \to \infty} (0) \leq \lim\limits_{x \to \infty} \dfrac{1 - \cos x}{x} \leq \lim\limits_{x \to \infty} \left(\dfrac{2}{x}\right) \rightarrow$

$0 \leq \lim\limits_{x \to \infty} \dfrac{1 - \cos x}{x} \leq 0 \rightarrow \lim\limits_{x \to \infty} \dfrac{1 - \cos x}{x} = 0$ .

d.) $g(x)=0 \rightarrow \frac{1-\cos x}{x}=0 \rightarrow 1-\cos x=0 \rightarrow$

$\cos x = 1 \rightarrow x = \pm 2\pi, \pm 4\pi, \pm 6\pi, \ldots$ .

$\boxed{74:26}$ $\lim_{x\to\infty} \sin\left(\frac{1}{x}\right) = \sin(0) = 0$ .

$\boxed{74:28}$ f) $\lim_{x\to 0^+} x \sin x = (0)\sin(0) = (0)(0) = 0$

g) $\lim_{x\to\infty} x \sin x$ does not exist since

$-1 \le \sin x \le +1$ so $x \sin x$ tends

toward $+\infty$ or $-\infty$ as $x \to \infty$ .