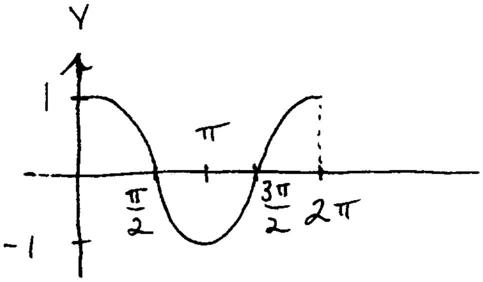
# HW #5

## Section 2.8

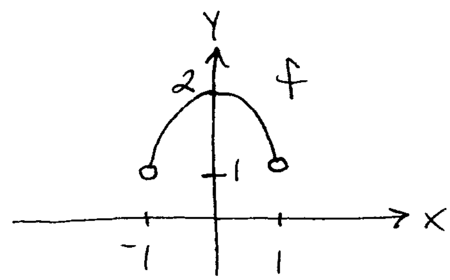



$\boxed{85:18}$ $Y = \cos X$

a.) on $[0, \frac{\pi}{2}]$: max. value $Y=1$ at $X=0$, min. value $Y=0$ at $X=\frac{\pi}{2}$.

b.) on $[0, 2\pi]$: max. value $Y=1$ at $X=0$ or $X=2\pi$, min. value $Y=-1$ at $X=\pi$.

$\boxed{85:19}$ $f(x) = \dfrac{x^3+x^4}{1+5x^2+x^6}$ on $[1,4]$; since $Y=x^4+x^3$ and $Y=1+5x^2+x^6$ are polynomials, they are continuous for all values of $x$, and since $Y=1+5x^2+x^6 \geq 1$ it is never equal to zero. Hence, $f$ is continuous on the closed interval $[1,4]$. It follows from the Maximum/Minimum Value Theorems that $f$ has a maximum value for some $x$ in $[1,4]$ and $f$ has a minimum value for some $x$ in $[1,4]$.




$\boxed{85:23}$ $f(x) = 2-x^2$ on $(-1,1)$

a.) $f$ has a maximum value of $Y=2$ at $X=0$.

b.) $f$ does NOT have a minimum value in $(-1,1)$.

$\boxed{85:26}$ Show that the equation $x^5 - 2x^3 + x^2 - 3x + 1 = 0$ has at least one solution in $[1,2]$.

Let $f(x) = x^5 - 2x^3 + x^2 - 3x + 1$ and $m = 0$. Function $f$ is continuous (since it's a polynomial) on the closed interval $[1,2]$. Since $f(1) = -2$, $f(2) = 15$, and $m = 0$ is between these values, it follows from the IMUT that there is at least one number $c$ in $[1,2]$ so that

$$f(c) = m, \quad \text{i.e.,}$$

$$c^5 - 2c^3 + c^2 - 3c + 1 = 0, \quad \text{i.e.,}$$

the original equation has at least one solution in $[1,2]$.

$\boxed{85:28}$ $f(x) = x^2 - 2x$ on $[-1,4]$ and $m = 5$: $f$ is continuous (since it's a polynomial) and $[-1,4]$ is a closed interval, so the conclusions of the IMUT follow, i.e., there is at least one number $c$ in $[-1,4]$ so that $f(c) = m = 5$ ($m=5$ is between $f(-1) = 3$ and $f(4) = 8$), i.e.,

$$c^2 - 2c = 5 \rightarrow c^2 - 2c - 5 = 0 \rightarrow$$

$$c = \frac{2 \pm \sqrt{4+20}}{2} = \frac{2 \pm 2\sqrt{6}}{2} = 1 \pm \sqrt{6} \rightarrow$$

$$c = 1 + \sqrt{6} \quad (c = 1 - \sqrt{6} \text{ is not in } [-1,4]!).$$

$\boxed{85:33}$ Is $x + \sin x = 1$ solvable?

Let $f(x) = x + \sin x$ and $m = 1$. Function $f$ is continuous since $y = x$ and $y = \sin x$ are continuous. Note that $f(0) = 0 + \sin 0 \rightarrow f(0) = 0$ and $f(\frac{\pi}{2}) = \frac{\pi}{2} + \sin\frac{\pi}{2} = \frac{\pi}{2} + 1$ with $m = 1$ between $f(0)$ and $f(\frac{\pi}{2})$. Consider $f$ on the closed interval $[0, \frac{\pi}{2}]$. It follows from the IMVT that there is at least one number $c$ in $[0, \frac{\pi}{2}]$ so that

$f(c) = m$, i.e.,

$c + \sin c = 1$, i.e.,

the original equation is solvable.

$\boxed{85:34}$ Is $x^3 = 2^x$ solvable?

If $x^3 = 2^x$ then $x^3 - 2^x = 0$ so let $f(x) = x^3 - 2^x$ and $m = 0$. Since $y = x^3$ and $y = 2^x$ are continuous for all values of $x$, it follows that $f$ is continuous for all values of $x$. Note that $f(1) = -1$ and $f(2) = 4$ with $m = 0$ between $f(1)$ and $f(2)$. Since $f$ is continuous on the closed interval $[1, 2]$, it follows from the IMVT that there is at least one number $c$ in $[1, 2]$ solving $f(c) = m$, i.e., $c^3 - 2^c = 0$, i.e., $c^3 = 2^c$.