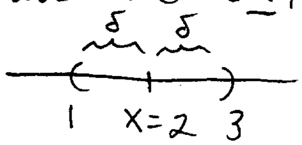
# HW #7

## Section 2.10

$\boxed{96:11}$ Find a $\delta > 0$ so that if $0 < |x-2| < \delta$ then $|x^2 - 4| < 1$ :

$|x^2-4| < 1$ iff $|(x-2)(x+2)| < 1$ iff $|x-2|\,|x+2| < 1$ ;
assume $\delta \le 1$ then $1 < x < 3$ and $3 < |x+2| < 5$



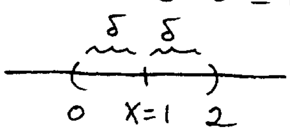
so that

$|x-2|\,|x+2| < |x-2|\,(5) < 1$

iff $|x-2|\,(5) < 1$ iff $|x-2| < \frac{1}{5}$ . Choose $\delta = \frac{1}{5}$ .

$\boxed{96:12}$ Find a $\delta > 0$ so that if $0 < |x-1| < \delta$ then $|x^2 + x - 2| < \frac{1}{2}$ :

$|x^2+x-2| < \frac{1}{2}$ iff $|(x-1)(x+2)| < \frac{1}{2}$ iff $|x-1|\,|x+2| < \frac{1}{2}$ ;
assume $\delta \le 1$ then $0 < x < 2$ and $2 < |x+2| < 4$

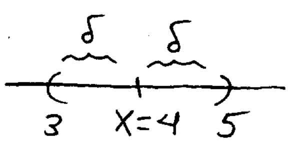


so that

$|x-1|\,|x+2| < |x-1|\,(4) < \frac{1}{2}$

iff $|x-1|\,(4) < \frac{1}{2}$ iff $|x-1| < \frac{1}{8}$ . Choose $\delta = \frac{1}{8}$ .

$\boxed{96:16}$ Show that if $0 < \delta < 1$ and $|x-4| < \delta$ then $|\sqrt{x} - 2| < \frac{\delta}{\sqrt{3}+2}$ : If $|x-4| < \delta < 1$ then



$3 < x < 5$ . Thus,

$$|\sqrt{x} - 2| = \left| \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}+2)} \right| = \frac{|x-4|}{\sqrt{x}+2} < \frac{\delta}{\sqrt{3}+2} .$$

$\boxed{96:27}$ Show that $\lim_{x \to 2} 3x = 5$ is false :

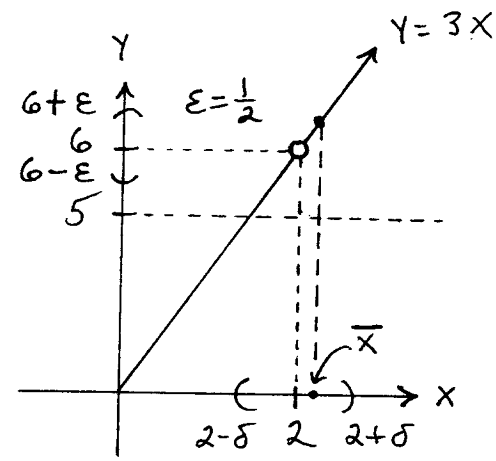
<u>If it were true</u> :

For each $\varepsilon > 0$ there is a $\delta > 0$ so that if $0 < |x-2| < \delta$ then $|3x-5| < \varepsilon$.

<u>Since it is false</u> :

There is an $\varepsilon > 0$ for which <u>no</u> $\delta > 0$ can be found so that if $0 < |x-2| < \delta$ then $|3x-5| < \varepsilon$.

Find such an $\varepsilon > 0$.




$\boxed{\text{Let } \varepsilon = \frac{1}{2}}$. Then for <u>any</u> $\delta > 0$ with $2-\delta < x < 2+\delta$, pick such an $\bar{x} > 2$ (See diagram). Then $0 < |\bar{x} - 2| < \delta$ for <u>any</u> $\delta > 0$

but $|f(\bar{x}) - 5| > 1$ (and hence $\not< \varepsilon = \frac{1}{2}$).

Thus, we have shown that $\lim_{x \to 2} 3x = 5$ is false.

## Section 3.1

$\boxed{111:17}$ $s(t) = t^3$

a.) distance $= s(2.1) - s(2) = 9.261 - 8 = 1.261$ ft.

b.) ave. vel. $= \dfrac{s(2.1)-s(2)}{2.1-2} = \dfrac{1.261}{0.1} = 12.61$ ft./sec.

c.) ave. vel. $= \dfrac{s(2+h)-s(2)}{(2+h)-2} = \dfrac{(2+h)^3 - 2^3}{h}$

$= \dfrac{\cancel{8} + 12h + 6h^2 + h^3 - \cancel{8}}{h} = \dfrac{\cancel{h}(12+6h+h^2)}{\cancel{h}} = 12+6h+h^2 \ \dfrac{\text{ft.}}{\text{sec.}}$

d.) vel. at $t=2$ is

vel. $= \lim_{h \to 0} (12+6h+h^2) = 12$ ft./sec.

$\boxed{111:24}$ $s(x) = x^2 \rightarrow s'(x) = 2x$

a.) magn. $= \dfrac{s(3.1)-s(3)}{3.1-3} = \dfrac{(3.1)^2 - 3^2}{0.1} = 6.1$

b.) magn. $= \dfrac{s(3.01)-s(3)}{3.01-3} = \dfrac{(3.01)^2 - 3^2}{0.01} = 6.01$

c.) magn. $= \dfrac{s(3.001)-s(3)}{3.001-3} = \dfrac{(3.001)^2 - 3^2}{0.001} = 6.001$

d.) magn. at 3 is $s'(3) = 6$

$\boxed{111:28}$ $s(x) = x^2$

a.) mass $= s(2.01) - s(2) = (2.01)^2 - 2^2 = 0.0401$ gm.

b.) density at 2 is approximately

$\dfrac{s(2.01)-s(2)}{2.01-2} = \dfrac{0.0401}{0.01} = 4.01$ gm./cm.

c.) density at 2 is approximately

$$\frac{s(2)-s(1.99)}{2-1.99} = \frac{0.0399}{0.01} = 3.99 \text{ gm./cm.}$$

d.) $$\frac{s(2+h)-s(2)}{(2+h)-2} = \frac{(2+h)^2-2^2}{h} = \frac{\cancel{4}+4h+h^2-\cancel{4}}{h} = \frac{\cancel{h}(4+h)}{\cancel{h}}$$

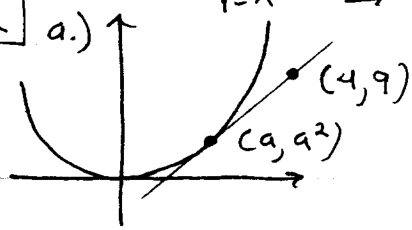
so density at 2 is $\lim_{h\to 0}(4+h) = 4$ gm./cm.

e.) $$\frac{s(2)-s(2+h)}{2-(2+h)} = \frac{2^2-(2+h)^2}{-h} = \frac{4-(4+4h+h^2)}{-h} = \frac{\cancel{4}-\cancel{4}-4h-h^2}{-h}$$

$= \dfrac{-\cancel{h}(4+h)}{-\cancel{h}}$ so density at 2 is

$\lim_{h\to 0}(4+h) = 4$ gm./cm.

$\boxed{11:42}$ a.) $Y=X^2 \to Y'=2X$




Slope of tangent line at $X=a$ is

$$2a = \frac{9-a^2}{4-a} \to 8a-2a^2 = 9-a^2 \to$$

$$a^2-8a+9=0 \to a = \frac{8\pm\sqrt{64-36}}{2} = 4\pm\sqrt{7} \text{ so } a=4-\sqrt{7}.$$

b.) If point is $(4,-9)$ then slope of tangent line at $x=a$ is

$$2a = \frac{-9-a^2}{4-a} \to 8a-2a^2 = -9-a^2 \to a^2-8a-9=0 \to$$

$$(a-9)(a+1)=0 \to a=9 \text{ or } a=-1 \text{ so } a=-1.$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

## Section 3.2

$\boxed{121:4}$ $f(x) = 3x - 1 \rightarrow$

$$f'(x) = \lim_{h \to 0} \frac{3(x+h) - 1 - (3x-1)}{h} = \lim_{h \to 0} \frac{\cancel{3x} + 3h \cancel{-1} \cancel{-3x} \cancel{+1}}{h}$$

$$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}} = 3$$

$\boxed{121:7}$ $f(x) = x^2 + 2x \longrightarrow$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2hx + h^2 + \cancel{2x} + 2h - \cancel{x^2} - \cancel{2x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 2)}{\cancel{h}} = 2x + 2$$

$\boxed{121:12}$ $f(x) = 7x^2 - \sqrt{x} \longrightarrow$

$$f'(x) = \lim_{h \to 0} \frac{7(x+h)^2 - \sqrt{x+h} - (7x^2 - \sqrt{x})}{h}$$

$$= \lim_{h \to 0} \frac{7(x^2 + 2hx + h^2) - 7x^2 + \sqrt{x} - \sqrt{x+h}}{h}$$

$$= \lim_{h \to 0} \left[ \frac{\cancel{7x^2} + 14hx + 7h^2 - \cancel{7x^2}}{h} + \frac{(\sqrt{x} - \sqrt{x+h})}{h} \frac{(\sqrt{x} + \sqrt{x+h})}{(\sqrt{x} + \sqrt{x+h})} \right]$$

$$= \lim_{h \to 0} \left[ \frac{\cancel{h}(14x + 7h)}{\cancel{h}} + \frac{x - (x+h)}{h(\sqrt{x} + \sqrt{x+h})} \right]$$

$$= \lim_{h \to 0} \left[ (14x + 7h) + \frac{\cancel{x} - \cancel{x} - \cancel{h}\ (-1)}{\cancel{h}(\sqrt{x} + \sqrt{x+h})} \right]$$

$$= 14x - \frac{1}{2\sqrt{x}}$$

$\boxed{121:16}$ $f(x) = \frac{1}{x^2} + x \longrightarrow$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} + (x+h) - \left(\frac{1}{x^2} + x\right)}{h}$$

$$= \lim_{h\to 0}\left[\frac{\frac{1}{x^2+2hx+h^2}-\frac{1}{x^2}}{\frac{h}{1}} + \frac{h}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{(\cancel{x^2}-\cancel{x^2}-2hx-h^2)}{(x^2+2hx+h^2)x^2}\frac{1}{h} + 1\right]$$

$$= \lim_{h\to 0}\left[\frac{\cancel{h}(-2x-h)}{\cancel{h}\cdot(x^2+2hx+h^2)\cdot x^2} + 1\right]$$

$$= \frac{-2x}{x^2x^2}+1 = \frac{-2}{x^3}+1 .$$

$\boxed{121:17}$ $f(x)=x^4 \to f'(x)=4x^3 \to f'(-1)=-4$

$\boxed{121:19}$ $f(x)=x^5 \to f'(x)=5x^4 \to f'(a)=5a^4$

$\boxed{121:21}$ $f(t)=t^{1/3} \to f'(t)=\frac{1}{3}t^{-2/3}=\frac{1}{3t^{2/3}} \to$

$f'(-8)=\frac{1}{3(-8)^{2/3}}=\frac{1}{3(4)}=\frac{1}{12}$

$\boxed{121:23}$ $f(x)=x^{\pi} \to f'(x)=\pi x^{\pi-1} \to f'(2)=\pi(2)^{\pi-1}$

$\boxed{121:25}$ $f(x)=x^{2/3} \to f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3x^{1/3}} \to$

$f'(8)=\frac{2}{3(8)^{1/3}}=\frac{2}{3(2)}=\frac{1}{3}$

$\boxed{121:27}$ $f(x)=(x^5)^{\frac{1}{4}}=x^{5/4} \to f'(x)=\frac{5}{4}x^{1/4} \to$

$f'(16)=\frac{5}{4}(16)^{1/4}=\frac{5}{4}(2)=\frac{5}{2}$

$\boxed{121:29}$ $f(x)=x^{-3} \to f'(x)=-3x^{-4}=\frac{-3}{x^4} \to$

$f'(2)=\frac{-3}{16}$.

$\boxed{121:34}$ $s(t)=t^4 \to s'(t)=4t^3$

a.) ave. vel. $= \frac{s(2.01)-s(2)}{2.01-2} = 32.24$ ft./sec.

b.) ave. vel. $= \dfrac{s(2) - s(1.99)}{2 - 1.99} = 31.76$ ft./sec.

c.) vel. at $t=2$ is $s'(2) = 32$ ft./sec.

$\boxed{121:35}$ $f(x) = x^4 \rightarrow f'(x) = 4x^3$

a.) magn. $= \dfrac{f(1.01) - f(1)}{1.01 - 1} = 4.0604$

b.) magn. at $x=1$ is $f'(1) = 4$

$\boxed{121:36}$ $f(x) = x^3 \rightarrow f'(x) = 3x^2$

a.) ave. density $= \dfrac{f(2.01) - f(2)}{2.01 - 2} = 12.06$ gm./cm.

b.) ave. density $= \dfrac{f(2.01) - f(1.99)}{2.01 - 1.99} = 12.0001$ gm./cm.

c.) density at $x=2$ is $f'(2) = 12$ gm./cm.

$\boxed{121:37}$ a.) $Y = x^3 \rightarrow Y' = 3x^2 \rightarrow$ slope $m = 3$ at $(1,1) \rightarrow$ tangent line is $Y = mX + b \rightarrow (1) = (3)(1) + b \rightarrow b = -2 \rightarrow$ $Y = 3X - 2$ ; check pt. $(2,4) \rightarrow 4 = 3(2) - 2$ so pt. $(2,4)$ is on line $Y = 3X - 2$.

b.)