HW # 9

Section 3.4

141:6 $D\left(x^6+5x^2+2\right)=6x^5+10x$

141:8 $D\left(2x^3+3x^{1/2}\right)=6x^2+3\cdot\frac{1}{2}x^{-1/2}$

141:14 $D\left\{(2x^2-1)(x^2-3)\right\}=(2x^2-1)(2x)+(4x)(x^2-3)$

141:20 $D\ \frac{1}{6}\left(7x-x^{3/2}\right)=\frac{1}{6}\left(7-\frac{3}{2}x^{1/2}\right)$

141:30 $D\ \frac{1}{(x+x^{1/2})}=\frac{(x+x^{1/2})(0)-(1)\left(1+\frac{1}{2}x^{-1/2}\right)}{(x+x^{1/2})^2}$

141:34 $D\ \frac{(2x+9)(3x^2-x)}{x^2}=D\ \frac{6x^3+25x^2-9x}{x^2}$

$$=\frac{(x^2)(18x^2+50x-9)-(6x^3+25x^2-9x)(2x)}{x^4}$$

141:38 $y=\frac{1}{2x+1}\rightarrow y'=\frac{(2x+1)(0)-(1)(2)}{(2x+1)^2}=\frac{-2}{(2x+1)^2}$

at $x=2\rightarrow$ slope $m=\frac{-2}{25}$ so tangent line is

$y=mx+b\rightarrow \frac{1}{5}=\frac{-2}{25}(2)+b\rightarrow b=\frac{9}{25}\rightarrow y=\frac{-2}{25}x+\frac{9}{25}$.

141:40 $y=\frac{x+1}{x+2}\rightarrow y'=\frac{(x+2)(1)-(x+1)(1)}{(x+2)^2}=\frac{1}{(x+2)^2}$

at $x=-1\rightarrow$ slope $m=1$ so tangent line is

$y=mx+b\rightarrow 0=(1)(-1)+b\rightarrow b=1\rightarrow y=x+1$.

141:41 $s(t)=2t^4+t^3+2t$ so velocity is

$s'(t)=8t^3+3t^2+2$ and $s'(1)=13$ units/sec.

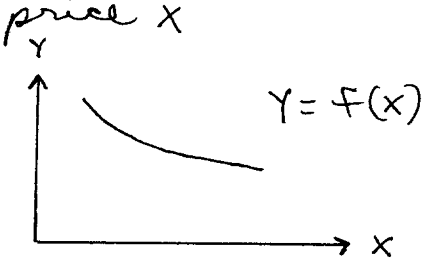
141:43 $s(x)=x^{1/3}$ so magnification is

$s'(x)=\frac{1}{3}x^{-2/3}=\frac{1}{3x^{2/3}}$ and $s'(8)=\frac{1}{3(4)}=\frac{1}{12}$

$\boxed{141:45}$ $s(x) = x \cdot x^{1/3} = x^{4/3}$ so density is $s'(x) = \frac{4}{3} x^{1/3}$ and $s'(8) = \frac{4}{3}(2) = \frac{8}{3}$ units/cm.

$\boxed{141:55}$ $(fgh)' = ((fg)h)' = (fg)\cdot h' + (fg)'h$
$= fgh' + (f'g + fg')h = f'gh + fg'h + fgh'$

$\boxed{141:56}$ $Y = f(x)$ is demand at price x



a.) In general, if X increases, then demand Y decreases. Thus, Y' is negative for particular values of X.

b.) If Y is gallons and X is cents, then
$Y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ is gal./cents

c.) $\frac{\left(\frac{\Delta Y}{Y}\right) \frac{\cancel{gal}}{\cancel{gal}}}{\left(\frac{\Delta X}{X}\right) \frac{\cancel{¢}}{\cancel{¢}}} = \frac{\left(\frac{\Delta Y}{Y}\right)}{\left(\frac{\Delta X}{X}\right)}$ has no units of measure

d.) $\varepsilon = \lim_{\Delta X \to 0} \frac{\Delta Y / Y}{\Delta X / X} = \lim_{\Delta X \to 0} \left(\frac{\Delta Y}{\Delta X}\right) \cdot \frac{X}{Y}$

$= \lim_{\Delta X \to 0} \frac{f(X + \Delta X) - f(X)}{\Delta X} \cdot \frac{X}{Y} = f'(X) \cdot \frac{X}{Y} = Y' \cdot \frac{X}{Y}$

e.) $\varepsilon \approx \frac{\Delta Y / Y}{\Delta X / X} = \frac{-2\%}{+1\%} = -2$

f.) $\varepsilon \approx \frac{\Delta Y / Y}{\Delta X / X} = \frac{-1\%}{+2\%} = \frac{-1}{2}$

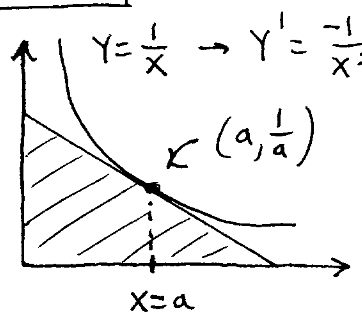
g.) $|\varepsilon| > 1$ <u>elastic</u> : changes in price lead to <u>larger changes</u> in demand

$|\varepsilon| < 1$ <u>inelastic</u> : changes in price lead to <u>smaller changes</u> in demand

h.) $Y = X^{-3} \rightarrow Y' = -3X^{-4} \rightarrow$ elasticity

$$\varepsilon = \left(\frac{X}{Y}\right) Y' = \frac{X}{X^{-3}} \cdot -3X^{-4} = -3X^{4}X^{-4} = -3 \cdot X^{0} = -3$$

$\boxed{141:57}$



Equation of tangent line is

$$Y = mX + b \rightarrow \frac{1}{a} = \frac{-1}{a^2}(a) + b \rightarrow$$

$$b = \frac{2}{a} \rightarrow Y = \frac{-1}{a^2}X + \frac{2}{a},$$

then

X-int : $Y = 0 \rightarrow X = 2a$

Y-int : $X = 0 \rightarrow Y = \frac{2}{a}$

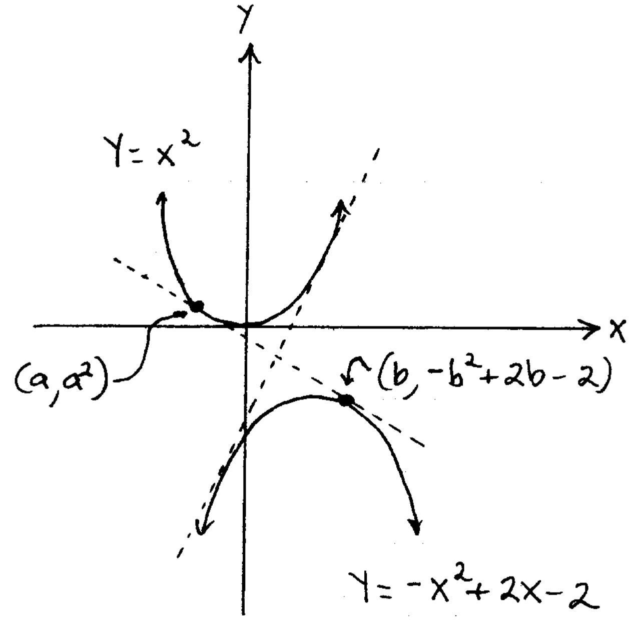
so area of triangle is $\frac{1}{2}(2a)\left(\frac{2}{a}\right) = 2$.

$\boxed{141:58}$ $Y = X^2 \rightarrow Y' = 2X$

so equation of tangent line at $x = a$ is

$Y - a^2 = 2a(X - a) \rightarrow$

$$\boxed{Y = 2aX - a^2} \; ;$$



$Y = -X^2 + 2X - 2 \rightarrow Y' = -2X + 2$ so equation of tangent line at $X = b$ is

$Y - (-b^2 + 2b - 2) = (-2b + 2)(X - b) \rightarrow$

$$\boxed{Y = (-2b + 2)X + (b^2 - 2)} \; .$$

Since these two equations represent the same line, slopes and Y-intercepts are equal :

$$\left.\begin{array}{l} 2a = -2b + 2 \\ -a^2 = b^2 - 2 \end{array}\right\} \quad \begin{array}{l} a = 1 - b \\ -(1-b)^2 = b^2 - 2 \rightarrow \end{array}$$

$-(1 - 2b + b^2) = b^2 - 2 \rightarrow 2b^2 - 2b - 1 = 0 \rightarrow$

$$b = \frac{2 \pm \sqrt{4+8}}{4} = \frac{1 \pm \sqrt{3}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$$ and

$a = \frac{1}{2} \mp \frac{\sqrt{3}}{2}$ so the two lines are

$$Y = (1 + \sqrt{3})X - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)^2 \text{ and}$$

$$Y = (1 - \sqrt{3})X - \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2 .$$

$\boxed{141:58}$ $Y = x^2 \xrightarrow{D} Y' = 2X$ so slope of line tangent at $X = a$ is $\boxed{2a}$;

ALTERNATE SOLUTION; See graph on previous page.

$y = -x^2 + 2X - 2 \rightarrow$
$y' = -2X + 2$ so
slope of line tangent at $X = b$ is $\boxed{-2b+2}$;

slope of line tangent at $X = a$ and $X = b$ is
$$\frac{\text{rise}}{\text{run}} = \frac{a^2 - (-b^2 + 2b - 2)}{a - b} = \boxed{\frac{a^2 + b^2 - 2b + 2}{a - b}} ;$$

Thus, $2a = -2b + 2 \rightarrow \boxed{a = 1 - b}$ and
$$\boxed{2a = \frac{a^2 + b^2 - 2b + 2}{a - b}} \rightarrow 2a^2 - 2ab = a^2 + b^2 - 2b + 2 \rightarrow$$
$$a^2 - 2ab = b^2 - 2b + 2 \rightarrow$$
$$(1-b)^2 - 2(1-b)b = b^2 - 2b + 2 \rightarrow \cdots \rightarrow$$
$$2b^2 - 2b - 1 = 0 \rightarrow \cdots \rightarrow b = \frac{1 \pm \sqrt{3}}{2} ;$$

then $a = 1 - b = \frac{1 \mp \sqrt{3}}{2} = a$, and tangent lines are:

$$y - \left(\frac{1+\sqrt{3}}{2}\right)^2 = (1+\sqrt{3})\left(X - \left(\frac{1+\sqrt{3}}{2}\right)\right)$$
and
$$y - \left(\frac{1-\sqrt{3}}{2}\right)^2 = (1-\sqrt{3})\left(X - \left(\frac{1-\sqrt{3}}{2}\right)\right) .$$