

Math 21A Kouba Practice Exam 1

1.) (8 pts.) Write the volume V of a sphere as a function of its surface area S .

2.) (8 pts.) Determine the domain for $f(x) = \frac{1}{2 - \sqrt{x^2 - 1}}$

3.) (8 pts.) Let $f(x) = \frac{x}{x+1}$. Find a function $g(x)$ so that $f(g(x) - 1) = x$.

4.) (8 pts. each) Evaluate the following limits.

a.) $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 5x + 6}$

b.) $\lim_{x \rightarrow -2} \frac{\sqrt{6+x} - 2}{x+2}$

c.) $\lim_{x \rightarrow \infty} \sin 3x$

d.) $\lim_{x \rightarrow 0} \frac{2x - \tan x}{x}$

e.) $\lim_{x \rightarrow 1^+} \frac{x^3 - 2x}{x-1}$

f.) $\lim_{x \rightarrow \infty} \frac{2 + \cos x}{x^2}$ (HINT : Use the Squeeze Principle.)

5.) (10 pts.) Use limits to determine the values of the constants A and B so that the following function is continuous for all values of x .

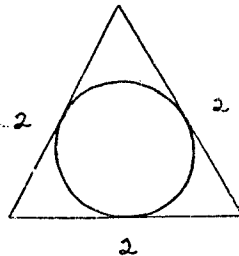
$$f(x) = \begin{cases} (Ax)^2 + Bx, & \text{if } x < -1 \\ 2, & \text{if } -1 \leq x \leq 2 \\ Ax - B + 3, & \text{if } x > 2. \end{cases}$$

6.) (9 pts.) Use the Intermediate Value Theorem to show that the equation $\frac{1}{x^2} = -2 + \sqrt{x}$ is solvable.

7.) (9 pts.) Give an ϵ, δ -proof for the following limit : $\lim_{x \rightarrow -2} (x^2 - x) = 6$.

Each of the following two EXTRA CREDIT PROBLEMS are worth 8 points. These problems are OPTIONAL.

1.) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 2x}$.



2.) Determine the radius of the circle inscribed in the given equilateral triangle of side length 2.