Math 21A

Exam 2 Solutions

1.) a.) $y' = x \cdot \frac{1}{2}(4-x^2)^{-1/2} \cdot -2x + (1) \cdot \sqrt{4-x^2}$

b.) $f'(x) = 10\left(\frac{7-x}{5x+3}\right)^9 \cdot \frac{(5x+3)(-1)-(7-x)(5)}{(5x+3)^2}$

c.) $y' = 4 \cdot 2\sin(x^3) \cdot \cos(x^3) \cdot 3x^2$

d.) $g'(x) = 0 - \sin(\tan(\sec(\frac{3}{x}))) \cdot \sec^2(\sec(\frac{3}{x})) \cdot \sec(\frac{3}{x})\tan(\frac{3}{x}) \cdot \frac{-3}{x^2}$

2.) $f'(x) = \lim_{h\to 0} \frac{\sqrt{4+(x+h)^2} - \sqrt{4+x^2}}{h} \cdot \frac{\sqrt{4+(x+h)^2} + \sqrt{4+x^2}}{\sqrt{4+(x+h)^2} + \sqrt{4+x^2}}$

$= \lim_{h\to 0} \frac{\cancel{4}+\cancel{x^2}+2hx+h^2-\cancel{4}-\cancel{x^2}}{h(\sqrt{4+(x+h)^2} + \sqrt{4+x^2})}$

$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}(\sqrt{4+(x+h)^2} + \sqrt{4+x^2})} = \frac{\cancel{2}x}{\cancel{2}\sqrt{4+x^2}} = \frac{x}{\sqrt{4+x^2}}$

3.) a.) $s'' = -32 \to s' = -32t + c \quad (t=0,\ s' = -64) \to$

$-64 = 0 + c \to c = -64 \to \boxed{s'(t) = -32t - 64}$;

$s = -16t^2 - 64t + c \quad (t = 0,\ s = 80) \to$

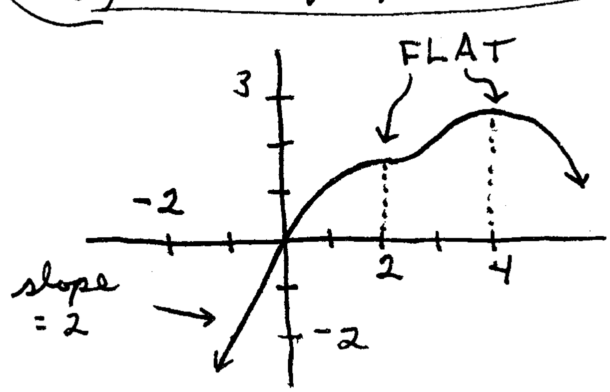
$80 = 0 - 0 + c \to c = 80 \to \boxed{s(t) = -16t^2 - 64t + 80}$

b.) strike ground: $s(t) = 0 \to -16t^2 - 64t + 80 = 0 \to$

$-16(t^2 + 4t - 5) = -16(t-1)(t+5) = 0 \to$ $t = 1$ sec.

c.) $s'(1) = -96$ ft./sec.

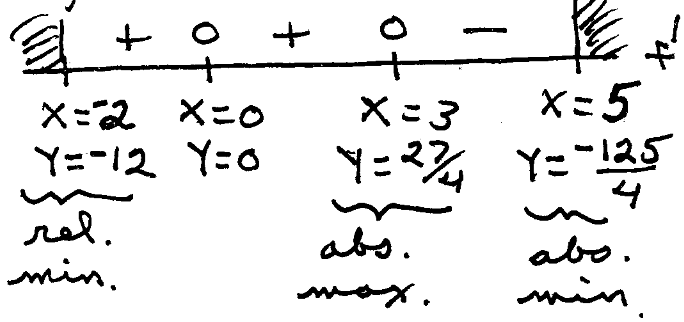
4.)



5.) $f(x) = \frac{1}{4}x^3(4-x)$ on $[-2, 5]$:

$x=0$: $y=0$ and $y=0$: $x=0$, $x=4$

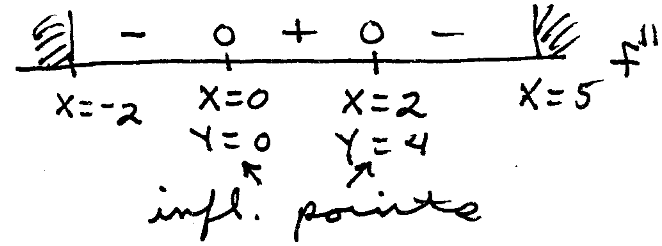
$f'(x) = 3x^2 - x^3 = x^2(3-x) = 0$



f is $\uparrow$ for $-2<x<0$, $0<x<3$,

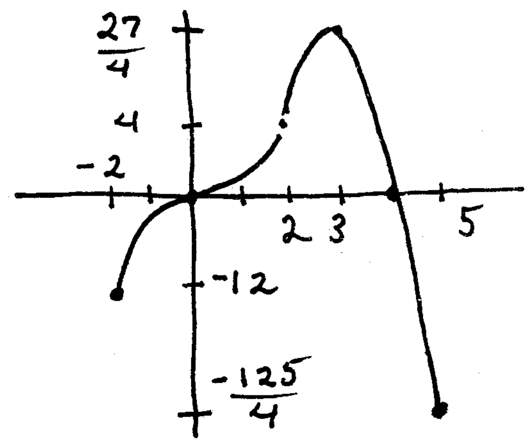
f is $\downarrow$ for $3<x<5$

$f''(x) = 6x - 3x^2 = 3x(2-x) = 0$



f is $\cup$ for $0<x<2$,

f is $\cap$ for $-2<x<0$, $2<x<5$



6.) a.) If function f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists at least one number c, $a<c<b$ satisfying

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

b.) $f(x) = x + \sqrt{x-2}$ is continuous on $[2, 6]$ since it is the sum of continuous functions; $f'(x) = 1 + \frac{1}{2\sqrt{x-2}}$ so f is differentiable on $(2, 6)$. Then by MVT

$$f'(c) = \frac{f(6)-f(2)}{6-2} \rightarrow 1 + \frac{1}{2\sqrt{c-2}} = \frac{8-2}{4} \rightarrow$$

$$1 + \frac{1}{2\sqrt{c-2}} = \frac{3}{2} \rightarrow \frac{1}{2\sqrt{c-2}} = \frac{1}{2} \rightarrow c = 3$$

7.) $s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$, where $s(t+h) - s(t)$ is distance in miles and h is time in hours, so units for $s'(t)$ are miles per hour (velocity)

8.) a.) \$140,000

b.) 4th : \$120,000
1st : \$60,000 } 1st → 4th : \$60,000

c.) RATE is slope of tangent line

at $t=1$: RATE $\approx \frac{\$100{,}000 - \$30{,}000}{2} = \$35{,}000$/month

EXTRA CREDIT :

1.) f has rel. max. at $x = 2$? :
CHECK : $f'(2) = 0$ and $f''(2) < 0$ (by Second Derivative Test) →

$f(x) = Ax^3 + B(x-2)^2 + 12x$ →
$f'(x) = 3Ax^2 + 2B(x-2) + 12$ then $f'(2) = 0$ →
$12A + 12 = 0$ → $\boxed{A = -1}$;

$f''(x) = 6Ax + 2B$ then $f''(2) < 0$ →

$12A + 2B < 0$ →
$12(-1) + 2B < 0$ →
$2B < 12$ → $\boxed{B < 6}$.

2.) $f(x) = \cos x \longrightarrow$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \left[\cos x \cdot \frac{\cos h - 1}{h} - \sin x \cdot \frac{\sin h}{h} \right]$$

$$= \cos x \cdot (0) - \sin x \cdot (1)$$

$$= -\sin x$$