

Math 21A Kouba

Practice Exam 3

1.) (7 pts. each) Determine  $y' = \frac{dy}{dx}$ . DO NOT SIMPLIFY ANSWERS.

a.)  $y = \ln(\tan x) + \log(5x + 3)$

b.)  $yx = x^y$  ( HINT: Apply a logarithm first. )

2.) (10 pts.) An initial deposit of \$1000 in a savings account grows to \$3000 in exactly 10 years. If interest is compounded weekly. What is the annual interest rate  $r$  ?

3.) (11 pts.) You are to construct an open (no top) rectangular box with a square base and a fixed volume of  $32 \text{ ft.}^3$ . What dimensions will result in a box of minimum surface area ?

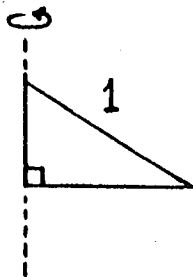
4.) (11 pts.) Assume that  $y$  is a function of  $x$ . Find an equation of the line perpendicular to the graph of  $y + \sin y = x - 1$  at  $y = 0$ .

5.) The equation  $x^5 + 2x = -5$  has exactly one solution  $r$ .

a.) (8 pts.) Use Newton's method to create a formula for creating successive approximations to  $r$ .

b.) (2 pts.) Let  $x_1 = 0$ . Use your formula in part a.) to compute  $x_2$  and  $x_3$ .

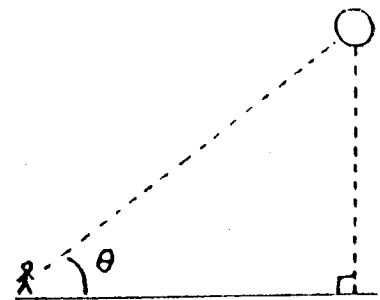
6.) (11 pts.) A right circular cone is formed by rotating a right triangle with hypotenuse 1 around one of its legs. Find the dimensions of the triangle which will result in the cone of largest volume ? ( Volume of cone is  $V = \frac{1}{3}\pi r^2 h$  . )



7.) (11 pts.) The area of a circular pool of oil is increasing at the rate of  $300\pi$  in.<sup>2</sup>/min. At what rate is the radius  $r$  of the pool changing when  $r = 75$  in. ?

8.) (10 pts.) Use differentials to estimate the value of  $\sqrt{102}$ .

9.) (12 pts.) A balloon, sitting 300 ft. away from an observer, begins rising vertically at the constant rate of 10 ft./sec. At what rate is the balloon's angle of elevation  $\theta$  (relative to the observer) changing after rising for 40 seconds ?



Each of the following two **EXTRA CREDIT PROBLEMS** is worth 10 points. These problems are **OPTIONAL**.

1.) Find the height  $h$  and radius  $r$  of the cylinder of maximum volume which can be inscribed inside a sphere of radius 1.

2.) Evaluate  $\lim_{n \rightarrow -\infty} \left( \frac{n}{n-3} \right)^{4n+7}$ .