Math 21A

Exam 3 Solutions

1.) a.) $Y' = \frac{1}{\tan x} \cdot \sec^2 x + \frac{1}{5x+3} \cdot 5 \cdot \log e$

b.) $YX = X^Y \rightarrow \ln(YX) = \ln X^Y \rightarrow$

$\ln X + \ln Y = Y \ln X \rightarrow \frac{1}{X} + \frac{1}{Y} Y' = Y \cdot \frac{1}{X} + Y' \cdot \ln X \rightarrow$

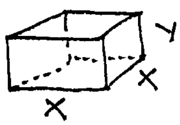
$\frac{1}{Y} Y' - Y' \ln X = \frac{Y}{X} - \frac{1}{X} \rightarrow Y' = \dfrac{\frac{Y}{X} - \frac{1}{X}}{\frac{1}{Y} - \ln X}$.

2.) $A = P\left(1 + \frac{r}{n}\right)^{nt} \rightarrow 3000 = 1000\left(1 + \frac{r}{52}\right)^{52(10)} \rightarrow$

$3 = \left(1 + \frac{r}{52}\right)^{520} \rightarrow 3^{\frac{1}{520}} = 1 + \frac{r}{52} \rightarrow$

$r = 52\left(3^{\frac{1}{520}} - 1\right) \approx 0.1099 \approx 11\%$

3.)



Volume $X^2 Y = 32 \rightarrow Y = \frac{32}{X^2}$,

minimize surface area

$A = X^2 + 4XY = X^2 + 4X\left(\frac{32}{X^2}\right) = X^2 + \frac{128}{X} \rightarrow$

$A' = 2X - \frac{128}{X^2} = \frac{2(X^3 - 64)}{X^2} = 0$ $\quad$ $-\ \ 0\ \ +$ $\quad A'$

$X = 4$ ft.

$Y = 2$ ft.

min. $A = 48$ ft.^{2}

4.) $Y + \sin Y = X - 1$ and $Y = 0$, $X = 1$ then

$Y' + \cos Y \cdot Y' = 1 \rightarrow Y' = \frac{1}{1 + \cos Y}$ at $Y = 0 \rightarrow Y' = \frac{1}{2}$

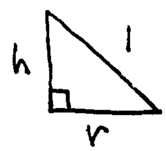
so slope $m = -2$ and line is $Y = -2(X - 1)$.

5.) a.) $f(x) = x^5 + 2x + 5 = 0 \rightarrow f'(x) = 5x^4 + 2 \rightarrow$

newton: $x_{n+1} = x_n - \dfrac{x_n^5 + 2x_n + 5}{5x_n^4 + 2} = \dfrac{5x_n^5 + \cancel{2x_n} - x_n^5 - \cancel{2x_n} - 5}{5x_n^4 + 2}$

$\rightarrow x_{n+1} = \dfrac{4x_n^5 - 5}{5x_n^4 + 2}$.

b.) $x_1 = 0$, $x_2 = \frac{-5}{2}$, $x_3 \approx -2.01$

6.) 

$r^2+h^2=1 \rightarrow r^2 = 1-h^2$, max. volume

$V=\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(1-h^2)h = \frac{1}{3}\pi(h-h^3) \rightarrow$

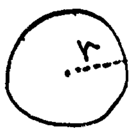
$V' = \frac{1}{3}\pi(1-3h^2) = 0 \rightarrow h = \frac{1}{\sqrt{3}}$

V': $+$ 0 $-$

$h = 1/\sqrt{3}$

$r = \sqrt{2/3}$

max. $V = \frac{2}{9\sqrt{3}}\pi$

7.) 

$\frac{dA}{dt} = 300\pi \ \frac{in.^2}{min.}$, find $\frac{dr}{dt}$ when

$r = 75$ in. $\rightarrow A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \rightarrow$

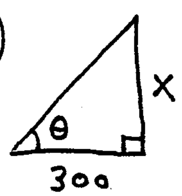
$300\cancel{\pi} = 2\cancel{\pi}(75)\cdot\frac{dr}{dt} \rightarrow \frac{dr}{dt} = 2$ in./min.

8.) $f(x) = \sqrt{x}$, $x: 100 \rightarrow 102$, $\Delta x = 2$

$f'(x) = \frac{1}{2\sqrt{x}}$ and $\Delta f \approx df \rightarrow$

$f(102) - f(100) \approx f'(100)\cdot\Delta x \rightarrow \sqrt{102} - 10 \approx \frac{1}{20}(2) \rightarrow$

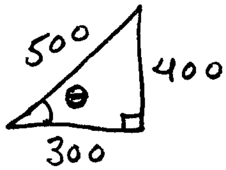
$\sqrt{102} \approx 10.1$

9.) 

$\frac{dx}{dt} = 10\ \frac{ft.}{sec.}$, find $\frac{d\theta}{dt}$ when

$t = 40$ sec. $(x = 400) \rightarrow$

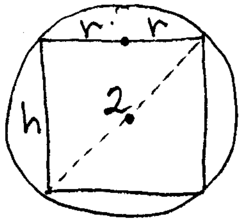
$\tan\theta = \frac{x}{300} \xrightarrow{D_t} \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{300}\cdot\frac{dx}{dt} \rightarrow$



$\frac{d\theta}{dt} = \frac{\cos^2\theta}{300}\cdot\frac{dx}{dt} = \frac{\left(\frac{3}{5}\right)^2}{300}\cdot(10) = 0.012\ \frac{\text{radians}}{\text{sec.}}$

Extra Credit

1.)



$$h^2 + (2r)^2 = 2^2 \rightarrow$$
$$h^2 + 4r^2 = 4 \rightarrow$$
$$r^2 = \frac{1}{4}(4 - h^2) \text{ , maximize}$$

volume $V = \pi r^2 h = \pi \frac{1}{4}(4-h^2)h = \frac{\pi}{4}(4h - h^3) \rightarrow$

$$V' = \frac{\pi}{4}(4 - 3h^2) = 0 \rightarrow h = \frac{2}{\sqrt{3}}$$

$+ \quad 0 \quad -$ V'

$h = 2/\sqrt{3}$

$r = \sqrt{\frac{2}{3}}$

2.) $$\lim_{n \to -\infty} \left(\frac{n}{n-3}\right)^{4n+7} = \lim_{n \to -\infty} \frac{1}{\left(\frac{n-3}{n}\right)^{4n+7}}$$

$$= \lim_{n \to -\infty} \frac{1}{\left(1 - \frac{3}{n}\right)^{4n}\left(1 - \frac{3}{n}\right)^7}$$

$$= \lim_{n \to -\infty} \frac{1}{\left(1 + \frac{1}{\left(\frac{-n}{3}\right)}\right)^{\left(\frac{-n}{3}\right)\cdot -4(3)} \cdot \left(1 - \frac{3}{n}\right)^7}$$

$$= \frac{1}{e^{-12} \cdot 1^7} = e^{12}$$