Math 21A Kouba
Practice Final Exam

1.) (10 pts. each) Diffferentiate each of the following. DO NOT SIMPLIFY answers.

a.) $y = e^{x^2} \cos^3(5x)$ b.) $y = \arcsin(2^x + \log x)$ c.) $y = x^{\ln x}$

2.) (10 pts.) You deposit \$2000 in a retirement account earning 12% annual interest compounded monthly. In how many years will the account grow to \$10,000 ?

3.) (10 pts.) The manager of the Economy Motel charges \$30 per room and rents 48 rooms each night. For each \$5 increase in room charge four (4) fewer rooms are rented. What charge per room will maximize the total amount of money the manager will make in one night ?

4.) (10 pts.) Use the limit definition of derivative to differentiate $f(x) = \dfrac{x^2}{x+1}$.

5.) (10 pts.) Use limits to determine the values of the constants A and B so that the following function is continuous for all values of x .

$$f(x) = \begin{cases} Bx^2 + Ax, & \text{if } x \le -1 \\ 2B - Ax, & \text{if } -1 < x \le 2 \\ x + 3, & \text{if } x > 2 . \end{cases}$$

6.) (10 pts.) Find all points (x, y) on the graph of $y = \dfrac{1}{x}$ with tangent lines passing through the point (4, 0).

7.) (10 pts.) The radius of a sphere is measured with an absolute percentage error of at most 4% . Use differentials to estimate the maximum absolute percentage error in computing the volume of the sphere. ($V = \frac{4}{3}\pi r^3$.)

8.) Consider the equation $27 - x^3 = \sin x$.

a.) (10 pts.) Use the Intermediate Value Theorem to verify that the equation is solvable.

b.) (5 pts.) Use Newton's method to estimate the value of the solution of the equation to three decimal places.

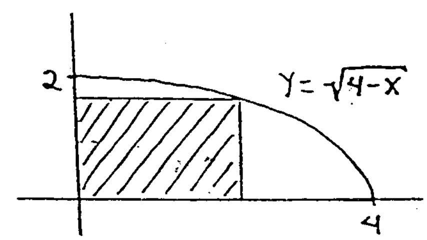
9.) Car B is 34 miles directly east of car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph.

a.) (10 pts.) At what rate is the distance between the cars changing after t $= \frac{1}{5}$ hr. ?

b.) (10 pts.) What is the minimum distance between the cars and at what time t does the minimum distance occur ?

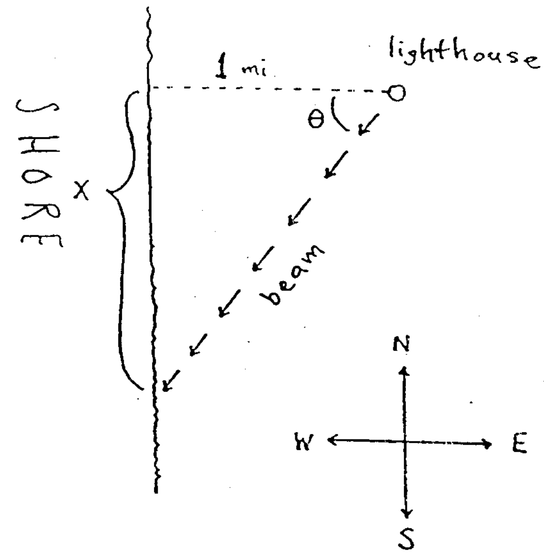
10.) (10 pts.) Find the slope and concavity of the graph of $xy + y^2 = 3X+1$ at the point (0, -1) .

11.) (10 pts.) Consider all rectangles in the first quadrant inscribed in such a way that their bases lie on the x-axis with the top corner on the graph of $y = \sqrt{4-x}$. Find the length and width of the rectangle of maximum area.



12.) (15 pts.) Consider the function $f(x) = x\, e^{\left(\frac{-x}{2}\right)}$. Determine where f is increasing, decreasing, concave up, and concave down. Identify all relative and absolute extrema, inflection points, x- and y-intercepts, and vertical and horizontal asymptotes. Sketch the graph. You may assume that $f'(x) = (1 - \frac{x}{2})\, e^{\left(\frac{-x}{2}\right)}$ and $f''(x) = (\frac{x}{4} - 1)\, e^{\left(\frac{-x}{2}\right)}$.

13.) (10 pts.) A lighthouse sits one (1) mile offshore with a light beam turning counter-clockwise at the rate of ten (10) revolutions per minute. How fast is the light beam racing down the shoreline when the beam strikes a point on the shore twelve miles south of the nearest point on the shore ? Write your final answer in MILES PER HOUR.



14.) (10 pts. each) Evaluate the following limits.

a.) $\lim_{x \to 0} \dfrac{x \sin x}{(\arctan x)^2}$

b.) $\lim_{x \to 0+} x \ln x$

c.) $\lim_{x \to \infty} (x^3 + 4)^{\frac{1}{x}}$

Each of the following three EXTRA CREDIT PROBLEMS is worth 10 points. These problems are OPTIONAL.

1.) Show that $\log_B C = \dfrac{\ln C}{\ln B}$.

2.) Find all values of K for which the the function $f(x) = -x^3 + Kx + x^2$ is NOT one-to-one.

3.) Find a tilted asymptote for the function $y = \sqrt{x^2 + x}$.