Math 21A
Kouba
Discussion Sheet 9

1.) Show that each function satisfies the assumptions of the Mean Value Theorem (MVT) over the given interval. Then determine all values of c guaranteed by the MVT.

a.) $f(x) = 3x^2 - 2x + 1$ on $[-1, 2]$ b.) $g(x) = \sqrt{9 - x^2}$ on $[-3, 0]$

c.) $f(x) = x^2(x - 1)$ on $[-1, 3]$ d.) $f(x) = x + \dfrac{1}{x}$ on $[1, 4]$

e.) $f(x) = x + \tan(x/2)$ on $[0, \pi/3]$ f.) $f(x) = \begin{cases} 2x^3 + 1\,, & \text{if } 0 \le x \le 1 \\ 1 - x^2\,, & \text{if } -1 \le x < 0 \end{cases}$

g.) $f(x) = \sin x + \cos x$ on $[0, \pi]$ h.) $f(x) = \dfrac{x + 1}{4 - x}$ on $[0, 3]$

2.) Consider the function $f(x) = 1 - x^{2/3}$ on the interval $[-1, 1]$. Show that $f(1) = f(-1) = 0$, but that $f'(x)$ is never zero on the interval [-1, 1]. Explain how this is possible in view of Rolle's Theorem.

3.) Let $f(x) = \begin{cases} 2 + x^2\,, & \text{if } 0 \le x \le 1 \\ 2x^3 + 1\,, & \text{if } 1 < x \le 2 \end{cases}$

a.) Sketch the graph of f .

b.) Show that f does NOT satisfy the assumptions of the Mean Value Theorem on the interval $[0, 2]$.

4.) Compute the following limits.

a.) $\lim\limits_{x \to 0} \dfrac{\sin 4x}{3x}$ b.) $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{xe^x - x}$ c.) $\lim\limits_{x \to 0} \dfrac{\sin 3x - 3x}{2x^2}$

d.) $\lim\limits_{x \to 0} \dfrac{(e^x - 1)^2}{\sin x^2}$ e.) $\lim\limits_{x \to 0} \dfrac{e^{-1/x^2}}{x}$ f.) $\lim\limits_{x \to \infty} \left(1 - \dfrac{3}{x}\right)^x$

g.) $\lim\limits_{x \to 0} (1 + 2x)^{5/x}$ h.) $\lim\limits_{x \to \infty} \left(\ln x\right)^{1/x}$ i.) $\lim\limits_{x \to 0} (x)^{\tan x}$

j.) $\lim\limits_{x \to \infty} \left(3^x + 4^x\right)^{1/x}$ k.) $\lim\limits_{x \to 0} \dfrac{2 \arcsin x}{\arctan 2x}$ l.) $\lim\limits_{x \to \infty} \dfrac{e^x \ln x}{e^{2x} - 2x + 1}$

m.) $\lim\limits_{x \to \infty} \dfrac{\ln 3x}{\log 2x}$ n.) $\lim\limits_{x \to 1} \dfrac{\log_3 x}{\log_5 x}$ o.) $\lim\limits_{x \to 0^+} \dfrac{4 \log_7 2x}{5 \log_2 3x}$

p.) $\lim\limits_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right)$ q.) $\lim\limits_{x \to 0^+} x^2 \ln x$ r.) $\lim\limits_{x \to 0^+} (e^x - 1) \ln x$

s.) $\lim\limits_{x \to \infty} \dfrac{(\ln x)^4}{x}$ t.) $\lim\limits_{x \to 0} \dfrac{\sin x^3}{(\sin x)^3}$ u.) $\lim\limits_{x \to \infty} e^{-x} \ln x$

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The following problem is for recreational purposes only.

5.) Two bicyclists are twelve miles apart. They begin riding toward each other, one pedaling at 4 mph and the other at 2 mph. At the same time a bumblebee begins flying back and forth between the riders at a constant speed of 10 mph. What is the total distance the bumblebee travels by the time the riders meet ?

"Try to be a rainbow in someone's cloud."- Maya Angelou