Section 2.1

1.) $f(x) = x^3 + 1$

a.) $ARC = \dfrac{f(3) - f(2)}{3 - 2} = \dfrac{28 - 9}{1} = 19$

b.) $ARC = \dfrac{f(1) - f(-1)}{1 - (-1)} = \dfrac{2 - 0}{2} = 1$

2.) $g(x) = x^2 - 2x$

a.) $ARC = \dfrac{g(3) - g(1)}{3 - 1} = \dfrac{3 - (-1)}{2} = \dfrac{4}{2} = 2$

b.) $ARC = \dfrac{g(4) - g(-2)}{4 - (-2)} = \dfrac{8 - 8}{6} = \dfrac{0}{6} = 0$

3.) $h(t) = \cot t$

a.) $ARC = \dfrac{h(\frac{3\pi}{4}) - h(\frac{\pi}{4})}{\frac{3\pi}{4} - \frac{\pi}{4}} = \dfrac{\cot(\frac{3\pi}{4}) - \cot(\frac{\pi}{4})}{\frac{2\pi}{4}}$

$= \dfrac{(-1) - (1)}{\frac{\pi}{2}} = -2 \cdot \dfrac{2}{\pi} = \dfrac{-4}{\pi}$

b.) $ARC = \dfrac{h(\frac{\pi}{2}) - h(\frac{\pi}{6})}{\frac{\pi}{2} - \frac{\pi}{6}} = \dfrac{\cot\frac{\pi}{2} - \cot\frac{\pi}{6}}{\frac{3\pi}{6} - \frac{\pi}{6}}$

$= \dfrac{0 - \sqrt{3}}{\frac{2\pi}{6}} = \dfrac{-\sqrt{3}}{\frac{\pi}{3}} = -\sqrt{3} \cdot \dfrac{3}{\pi} = \dfrac{-3\sqrt{3}}{\pi}$

4.) $g(t) = 2 + \cos t$

a.) $ARC = \dfrac{g(\pi) - g(0)}{\pi - 0} = \dfrac{(2+\cos\pi) - (2+\cos 0)}{\pi}$

$= \dfrac{\cancel{2} + (-1) - \cancel{2} - (1)}{\pi} = \dfrac{-2}{\pi}$

b.) $ARC = \dfrac{g(\pi) - g(-\pi)}{\pi - (-\pi)} = \dfrac{(2+\cos\pi) - (2+\cos(-\pi))}{\pi}$

$= \dfrac{\cancel{2} + (-1) - \cancel{2} - (-1)}{2\pi} = \dfrac{0}{2\pi} = 0$

5.) $R(\theta) = \sqrt{4\theta + 1}$,

$ARC = \dfrac{R(2) - R(0)}{2 - 0} = \dfrac{\sqrt{9} - \sqrt{1}}{2} = \dfrac{3-1}{2} = 1$

6.) $P(\theta) = \theta^3 - 4\theta^2 + 5\theta$

$ARC = \dfrac{P(2) - P(1)}{2-1} = \dfrac{2-2}{1} = \dfrac{0}{1} = 0$

15.) a.) PQ_1 : SLOPE $\approx \dfrac{650}{15} = \dfrac{130}{3} \approx 43.3$ m./sec.

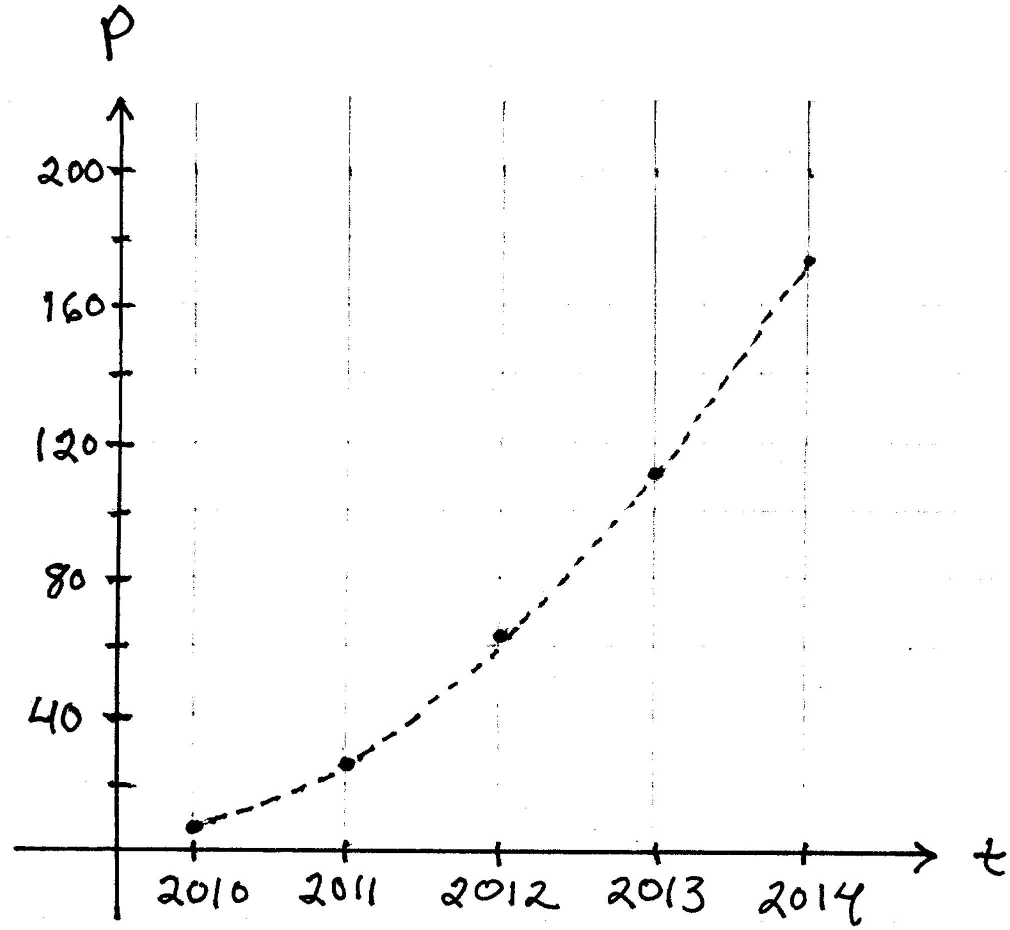
PQ_2: SLOPE $\approx \dfrac{650}{13} = 50$ m./sec.

PQ_3 : SLOPE $\approx \dfrac{650}{12.5} = 52$ m./sec.

PQ_3: SLOPE $\approx \dfrac{650}{12.6} \approx 51.6$ m./sec.

17.) a.)

years t	\$1000's profit p
2010	6
2011	27
2012	62
2013	111
2014	174



b.) $$ARC = \frac{P(2014) - P(2012)}{2014 - 2012}$$

$$= \frac{174 - 62}{2} = \frac{112}{2} = 56 \ \$1000\text{'s}/\text{yr.}$$

21.) a.) $[0,1]$:

$$ARC = \frac{S(1) - S(0)}{1 - 0} \approx \frac{15 - 0}{1} = 15 \text{ mi./hr.}$$

$[1, 2.5]$:

$$ARC = \frac{S(2.5) - S(1)}{2.5 - 1} \approx \frac{20 - 15}{1.5} = \frac{5}{3/2} = 5\left(\frac{2}{3}\right) = \frac{10}{3} \frac{\text{mi.}}{\text{hr.}}$$

$[2.5, 3.5]$:

$$ARC = \frac{S(3.5) - S(2.5)}{3.5 - 2.5} \approx \frac{30 - 20}{1} = 10 \text{ mi.}/\text{hr.}$$

22.) a.) $[0, 3]$:

$$ARC = \frac{A(3) - A(0)}{3 - 0} \approx \frac{10 - 15}{3} = \frac{-5}{3}$$

$$\approx -1.67 \text{ gal./day}$$

$[0, 5]$:

$$ARC = \frac{A(5) - A(0)}{5 - 0} \approx \frac{3.5 - 15}{5} = \frac{-11.5}{5}$$

$$= -2.3 \text{ gal./day}$$

$[7, 10]$:

$$ARC = \frac{A(10) - A(7)}{10 - 7} \approx \frac{0 - 1.5}{3} = \frac{-1.5}{3}$$

$$= -0.5 \text{ gal./day}$$

Math 21A
Kouba
Worksheet 1

1.) Determine the domain and range of each function.

a.) $f(x) = 9 - x^2$

b.) $f(x) = 3\cos 4x$

c.) $y = 5 + \sqrt{16 - x}$

d.) $g(x) = -\sqrt{625 - x^2}$

e.) $y = x + |x| + 3$

f.) $f(x) = \begin{cases} x^2 + 4, & \text{if } x < -1 \\ 3 - x^2, & \text{if } x \geq -1. \end{cases}$

2.) Determine the domain for $f(x) = \dfrac{7}{3 - \sqrt{x^2 - 16}}$.

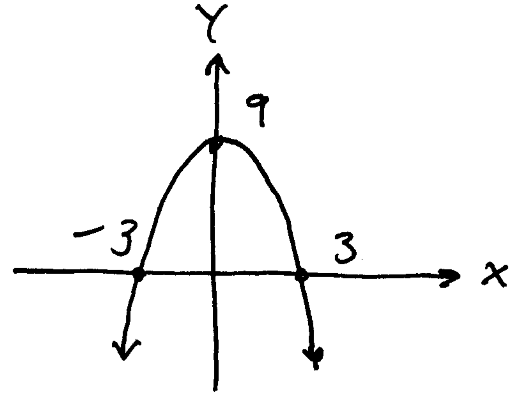
3.) Let $f(x) = \dfrac{x}{3 - x}$ and $g(x) = \dfrac{x - 2}{x + 4}$.

a.) Determine $(f \circ g)(x)$.

b.) Determine $(g \circ f)(x)$.

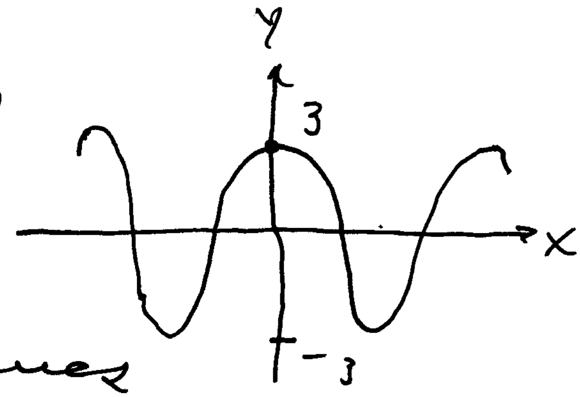
Worksheet 1

1.) a.) $f(x) = 9 - x^2$
(parabola)



Domain : all x-values
Range : $Y \leq 9$

b.) $f(x) = 3 \cos 4x$;



$-1 \leq \cos 4x \leq +1 \rightarrow$

$-3 \leq 3 \cos 4x \leq 3$;

Domain : all x-values

Range : $-3 \leq Y \leq +3$

c.) $y = 5 + \sqrt{16 - x}$; $16 - x \geq 0 \rightarrow$
$x \leq 16$ so

Domain : $x \leq 16$;

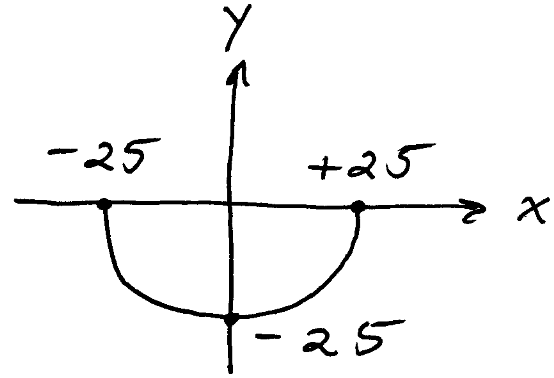
$0 \leq \sqrt{16 - x} < \infty$ so
$5 \leq 5 + \sqrt{16 - x} < \infty$,

Range : $y \geq 5$

d.) $y = -\sqrt{625 - x^2} \rightarrow$

$Y^2 = 625 - x^2 \rightarrow x^2 + Y^2 = 25^2$
(circle centered at (0,0)
with radius $r = 25$) ; so

$Y = -\sqrt{625 - x^2}$
is bottom half
of circle ;



Domain : $-25 \le x \le 25$

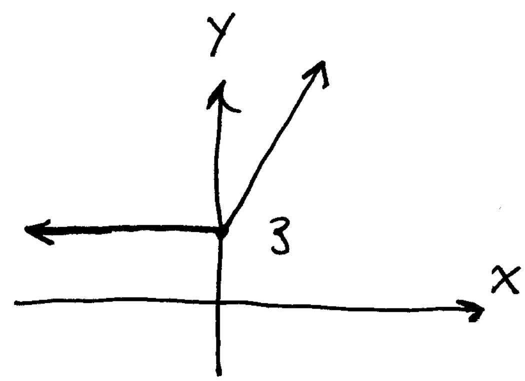
Range : $-25 \le Y \le 0$

e.) Recall : $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$;

$y = x + |x| + 3$

$= \begin{cases} x + (x) + 3 & \text{if } x \ge 0 \\ x + (-x) + 3 & \text{if } x < 0 \end{cases}$

$= \begin{cases} 2x + 3 & \text{if } x \ge 0 \\ 3 & \text{if } x < 0 \end{cases}$



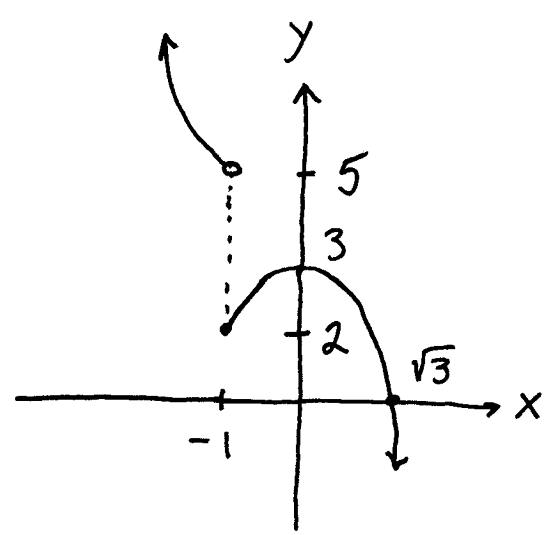
Domain : all x-values

Range : $Y \ge 3$

f.) $f(x) = \begin{cases} x^2 + 4 & \text{if } x < -1 \\ 3 - x^2 & \text{if } x \ge -1 \end{cases}$

Domain : all x-values

Range : $Y \le 3$, $Y > 5$



2.) $f(x) = \dfrac{7}{3-\sqrt{x^2-16}}$;

$x^2 - 16 = (x-4)(x+4) \geq 0$

sign chart : $+ \quad 0 \quad - \quad 0 \quad +$ at $x=-4$, $x=4$

so $x \geq 4$, $x \leq -4$; AND

$3 - \sqrt{x^2-16} \neq 0 \rightarrow 3 \neq \sqrt{x^2-16} \rightarrow$

$9 \neq x^2 - 16 \rightarrow x^2 \neq 25 \rightarrow x \neq \pm 5$;

Domain : $x \geq 4, x \leq -4$ and $x \neq \pm 5$

3.) $f(x) = \dfrac{x}{3-x}$, $g(x) = \dfrac{x-2}{x+4}$

a.) $(f \circ g)(x) = f(g(x))$

$$= f\left(\frac{x-2}{x+4}\right) = \frac{\left(\frac{x-2}{x+4}\right)}{3-\left(\frac{x-2}{x+4}\right)} \cdot \frac{x+4}{x+4}$$

$$= \frac{x-2}{3(x+4)-(x-2)}$$

$$= \frac{x-2}{3x+12-x+2} = \frac{x-2}{2x+14}$$

b.) $(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{x}{3-x}\right) = \frac{\left(\frac{x}{3-x}\right)-2}{\left(\frac{x}{3-x}\right)+4} \cdot \frac{3-x}{3-x}$$

$$= \frac{x - 2(3-x)}{x + 4(3-x)} = \frac{x - 6 + 2x}{x + 12 - 4x} = \frac{3x-6}{12-3x}$$

$$= \frac{3(x-2)}{3(4-x)} = \frac{x-2}{4-x}$$