

## Section 3.4

$$1.) \quad s = t^2 - 3t + 2 = (t-2)(t-1)$$
$$\text{vel. } s' = 2t - 3 = 2\left(t - \frac{3}{2}\right)$$
$$\text{acc. } s'' = 2 \quad ; \quad 0 \leq t \leq 2$$

$$a.) \quad \text{displ.} = s(2) - s(0) = 0 - 2 = -2 \text{ m.};$$
$$\text{ave. vel.} = \frac{s(2) - s(0)}{2 - 0} = \frac{-2}{2} = -1 \text{ m./s.}$$

$$b.) \quad \text{speed} = |s'(0)| = |-3| = 3 \text{ m./s.}$$
$$\text{speed} = |s'(2)| = |1| = 1 \text{ m./s.}$$
$$\text{acc.} = s''(0) = 2 \text{ m./s.}^2$$
$$\text{acc.} = s''(2) = 2 \text{ m./s.}^2$$

$$c.) \quad s'(t) = 2\left(t - \frac{3}{2}\right) = 0 \rightarrow$$
$$\text{vel. } s' = 0 \text{ m./s. when } \boxed{t = \frac{3}{2}} \text{ sec.}$$

$$\begin{array}{c} - \quad 0 \quad + \\ \hline t = \frac{3}{2} \end{array} \quad s'(t)$$

and object changes direction  
(moving left to moving right)

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$$4.) \quad S = \frac{1}{4}t^4 - t^3 + t^2 \\ = \frac{1}{4}t^2(t^2 - 4t + 4) = \frac{1}{4}t^2(t-2)^2,$$

$$\text{vel. } S' = t^3 - 3t^2 + 2t = t(t-1)(t-2),$$

$$\text{acc. } S'' = 3t^2 - 6t + 2 \quad ; \quad 0 \leq t \leq 3$$

$$a.) \text{ displ. } = S(3) - S(0) = \frac{9}{4} - 0 = \frac{9}{4} \text{ m.},$$

$$\text{ave. vel. } = \frac{S(3) - S(0)}{3 - 0} = \frac{\frac{9}{4}}{\frac{3}{1}} = \frac{9}{4} \cdot \frac{1}{3} = \frac{3}{4} \text{ m./s.}$$

$$b.) \text{ speed } = |S'(0)| = |0| = 0 \text{ m./s.}$$

$$\text{speed} = |S'(3)| = |6| = 6 \text{ m./s.},$$

$$\text{acc} = \mp S''(0) = 2 \text{ m./s.}^2,$$

$$\text{acc} = S''(3) = 11 \text{ m./s.}^2$$

$$c.) \text{ vel. } S' = t(t-1)(t-2) = 0 \rightarrow \\ t = 0, t = 1, \text{ or } t = 2 \text{ sec.}$$

$$\begin{array}{ccccccc} & - & 0 & + & 0 & - & 0 & + \\ & & | & & | & & | & \\ \hline & & t=0 & & t=1 & & t=2 & \\ & & & & & & & S'(t) \end{array}$$

$t=1$  sec.  $s'(1)=0$  and object changes direction (moving right to moving left); when  $t=2$  sec.  $s'(2)=0$  and object changes direction (moving left to moving right)

$$7.) \quad s = t^3 - 6t^2 + 9t = t(t-3)^2,$$

$$\text{vel. } s' = 3t^2 - 12t + 9$$

$$= 3(t^2 - 4t + 3) = 3(t-1)(t-3),$$

$$\text{acc. } s'' = 6t - 12 = 6(t-2)$$

$$a.) \text{ vel. } s' = 0 \rightarrow 3(t-1)(t-3) = 0 \rightarrow$$

$$\boxed{t=1} \text{ sec.}, \quad \boxed{t=3} \text{ sec.};$$

$$\text{acc. } s''(1) = -6 \text{ m./s.}^2,$$

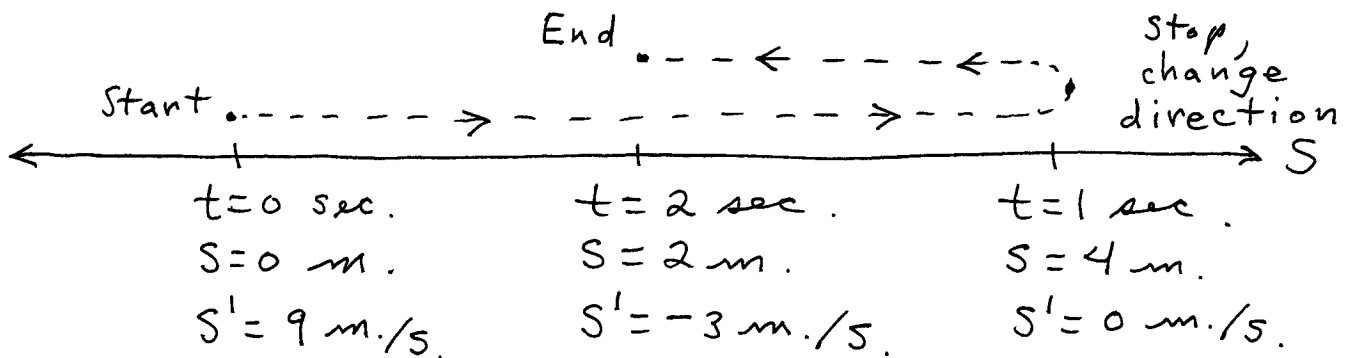
$$\text{acc. } s''(3) = 6 \text{ m./s.}^2$$

$$b.) \text{ acc. } s'' = 0 \rightarrow 6(t-2) = 0 \rightarrow \boxed{t=2} \text{ sec.}$$

$$\rightarrow \text{vel. } s'(2) = -3 \text{ m./s.}$$

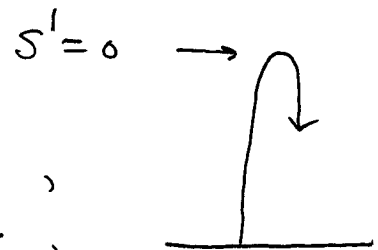
$$c.) \quad s(0) = 0 \text{ m. and } s'(0) = 9 \text{ m./s.}$$

so object starts at the origin and begins moving to the right;



the total distance traveled is

$$4 + 2 = 6 \text{ m.}$$



10.) a.)  $S = 24t - 0.8t^2 \text{ m.}$

vel.  $S' = 24 - 1.6t \text{ m./s.}$

acc.  $S'' = -1.6 \text{ m./s}^2$

b.) highest point:  $S' = 0 \rightarrow$

$$24 - 1.6t = 0 \rightarrow t = 15 \text{ sec.}$$

c.)  $S(15) = 24(15) - 0.8(15)^2 = 180 \text{ m.}$

d.) If  $S = 90 \text{ m.} \rightarrow 90 = 24t - \frac{4}{5}t^2 \rightarrow$

$$\frac{4}{5}t^2 - 24t + 90 = 0 \rightarrow 4t^2 - 120t + 450 = 0$$

$$\rightarrow 2(2t^2 - 60t + 225) = 0 \rightarrow$$

$$t = \frac{60 \pm \sqrt{3600 - 4(2)(225)}}{4} \approx \frac{60 \pm 42.4}{4} \rightarrow$$

$t \approx 4.4 \text{ sec.}$  or  $t \approx 25.6 \text{ sec.}$  (No)

e.)  $S = 0 \rightarrow 24t - 0.8t^2 = 0 \rightarrow$

$$t(24 - 0.8t) = 0 \rightarrow t = 0 \text{ sec. or}$$

$$24 - 0.8t = 0 \rightarrow \text{rock is aloft for}$$

$$\boxed{t = 30} \text{ sec.}$$

11.) ht.  $s = 15t - \frac{1}{2}gt^2$  has max.  $s$  when  $t = 20$  sec.; at max

$$s'(20) = 0 \rightarrow s' = 15 - \frac{1}{2}g \cdot 2t$$

$$= 15 - gt. \rightarrow s'(20) = 15 - g(20) = 0$$

$$\rightarrow 20g = 15 \rightarrow g = \frac{15}{20} = \frac{3}{4} \text{ m./sec.}^2$$

12.)  $s = 832t - 2.6t^2$  (moon)

$$h = 832t - 16t^2$$
 (earth)

a.)  $s = 0 \rightarrow 832t - 2.6t^2 = 0 \rightarrow$

$$t(832 - 2.6t) = 0 \rightarrow t = 0 \text{ or}$$

$$832 - 2.6t = 0 \rightarrow \text{bullet is aloft for}$$

$$\boxed{t = 320} \text{ sec. ; } s' = 0 \rightarrow 832 - 5.2t = 0$$

$$\rightarrow t = 160 \text{ sec. } \rightarrow \text{height of bullet}$$

$$\text{is } s(160) = \underline{66,560 \text{ ft.}}$$

b.)  $h = 0 \rightarrow 832t - 16t^2 = 0 \rightarrow$

$$t(832 - 16t) = 0 \rightarrow t = 0 \text{ or}$$

$$832 - 16t = 0 \rightarrow \text{bullet is aloft for}$$

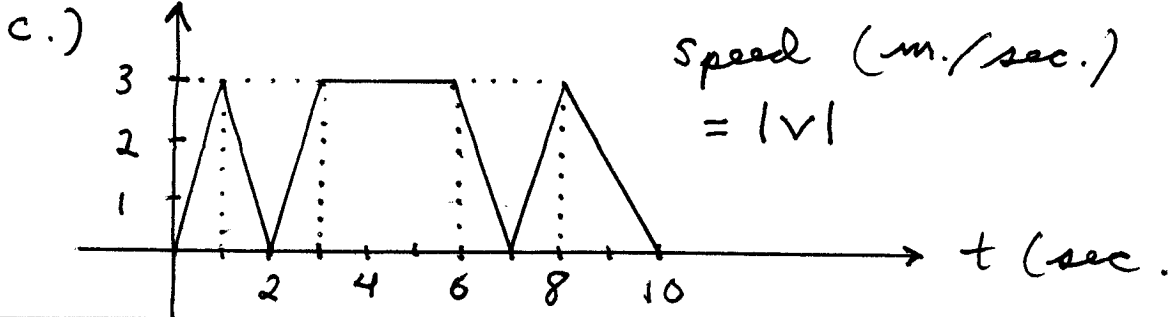
$$\boxed{t = 52} \text{ sec. ; } h' = 0 \rightarrow 832 - 32t = 0$$

$$\rightarrow t = 26 \text{ sec. } \rightarrow \text{height of bullet}$$

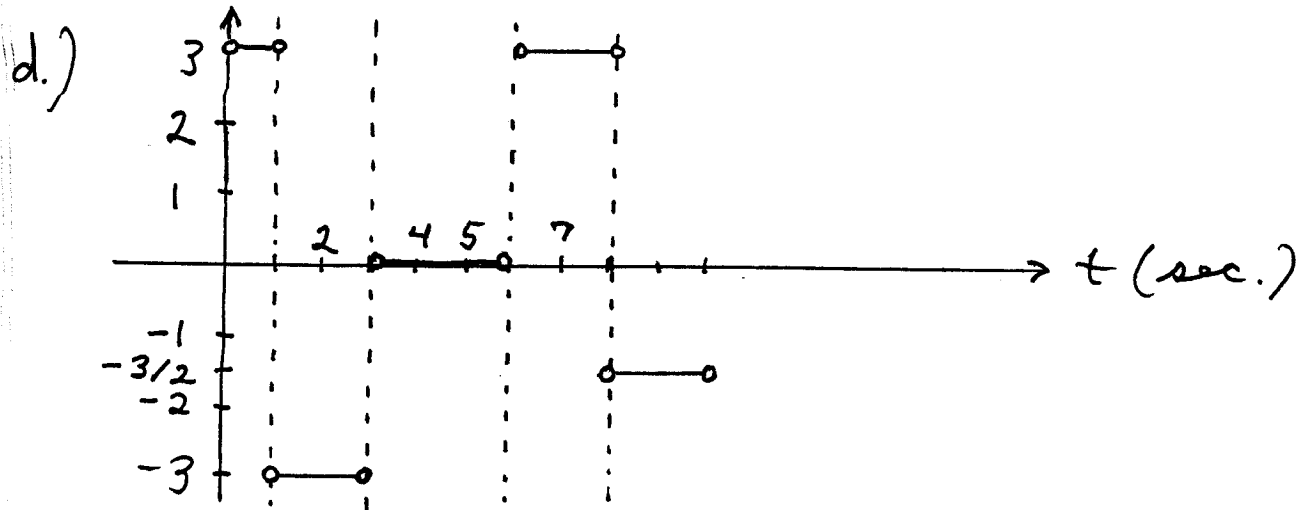
$$\text{is } h(26) = \underline{10,816 \text{ ft.}}$$

15.) a.)  $t = 2 \text{ sec.}, t = 7 \text{ sec.}$

b.)  $3 \leq t \leq 6 \text{ sec.}$



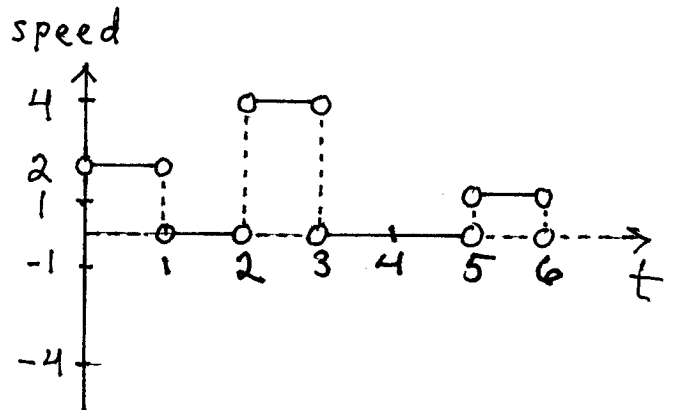
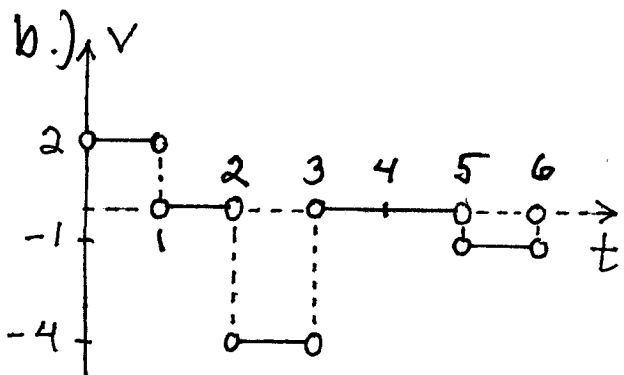
accel. :  $\frac{dv}{dt}$  (m./sec.<sup>2</sup>)

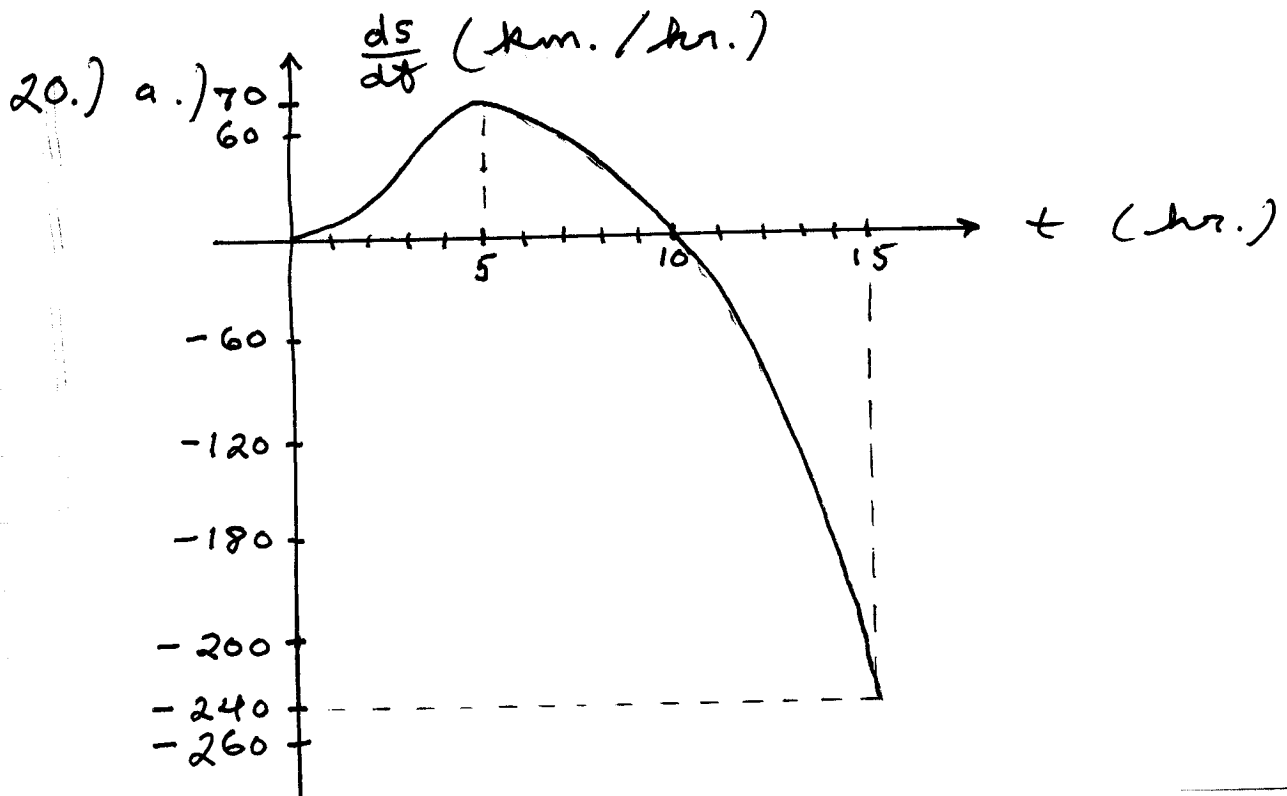


16.) a.) move left :  $2 \leq t \leq 3, 5 \leq t \leq 6$

move right :  $0 \leq t \leq 1$

stand still :  $1 \leq t \leq 2, 3 \leq t \leq 5$





21.)  $s: \textcircled{C}$ ,  $v = \frac{ds}{dt} : \textcircled{A}$ ,  $a = \frac{d^2s}{dt^2} : \textcircled{B}$

25.)  $b = 10^6 + 10^4 t - 10^3 t^2 \xrightarrow{D}$

$$\frac{db}{dt} = 10^4 - 10^3 \cdot 2t = 10,000 - 2000t$$

a.)  $t=0 \rightarrow \frac{db}{dt} = 10,000$  bact./hr.

b.)  $t=5 \rightarrow \frac{db}{dt} = 10,000 - 10,000 = 0$  bact./hr.

c.)  $t=10 \rightarrow \frac{db}{dt} = 10,000 - 20,000 = -10,000$  bact./hr.

26.)  $S = \frac{1}{60} \omega^{1/2} h^{1/2}$  and  $h = 180$  so

$$S = \frac{1}{60} \sqrt{180} \omega^{1/2} = \frac{1}{60} \sqrt{36 \cdot 5} \omega^{1/2} = \frac{6}{60} \sqrt{5} \omega^{1/2} \rightarrow$$

$$\frac{dS}{d\omega} = \frac{1}{10} \sqrt{5} \cdot \frac{1}{2} \omega^{-1/2} = \frac{\sqrt{5}}{20} \frac{1}{\sqrt{\omega}} ;$$

if  $\omega$  is "small",  $\frac{dS}{d\omega}$  is "large"

28.)  $Q(t) = 200(30-t)^2 = 200(t^2 - 60t + 900)$

$\xrightarrow{D} \frac{dQ}{dt} = 200(2t - 60)$  and  $t = 10$  min.  $\rightarrow$

$$\frac{dQ}{dt} = 200(2(10) - 60) = -8000 \text{ gal./min. ;}$$

$$\text{Average rate} = \frac{Q(10) - Q(0)}{10 - 0} = \frac{80,000 - 180,000}{10} \\ = -10,000 \text{ gal./min.}$$

29.)  $S = 1.1V + 0.054V^2 \xrightarrow{D}$

$$\frac{dS}{dV} = 1.1 + 0.108V ;$$

$$V = 35 : \frac{dS}{dV} = 1.1 + 0.108(35) = 4.88 \text{ ft./mph.}$$

$$V = 70 : \frac{dS}{dV} = 1.1 + 0.108(70) = 8.66 \text{ ft./mph. ;}$$

the larger that  $V$  is, the larger that rate  $\frac{dS}{dV}$  is



$$30.) a.) V = \frac{4}{3} \pi r^3 \xrightarrow{D}$$

$$\frac{dV}{dr} = \frac{4}{3} \pi \cdot 3r^2 = 4\pi r^2,$$

$$r=2 : \frac{dV}{dr} = 4\pi (2)^2 = 16\pi \text{ ft.}^3/\text{ft.}$$

$$31.) \text{ Distance } D = \frac{10}{9} t^2 \text{ m. } \rightarrow$$

$$\text{Velocity } v = \frac{dD}{dt} = \frac{20}{9} t \text{ m./sec. } j$$

$$\text{if } v = \frac{200 \text{ km.}}{1 \text{ hr.}} \cdot \frac{1 \text{ hr.}}{3600 \text{ sec.}} \cdot \frac{1000 \text{ m.}}{1 \text{ km}}$$

$$= \frac{500}{9} \text{ m./sec.}, \text{ then}$$

$$\frac{500}{9} = \frac{20}{9} t \rightarrow \boxed{t = 25} \text{ sec. and}$$

$$D = \frac{10}{9} (25)^2 \approx 694.4 \text{ m.}$$