Sec. 3.5

1.) $y = -10x + 3\cos x \xrightarrow{D} y' = -10 - 3\sin x$

2.) $y = \frac{3}{x} + 5\sin x \xrightarrow{D} y' = \frac{-3}{x^2} + 5\cos x$

3.) $y = x^2 \cos x \xrightarrow{D} y' = x^2 \cdot -\sin x + 2x \cdot \cos x$

8.) $g(x) = \frac{\cos x}{\sin^2 x} \xrightarrow{D}$

$$g'(x) = \frac{\sin^2 x \cdot -\sin x - \cos x(\sin x \cdot \cos x + \cos x \cdot \sin x)}{\sin^4 x}$$

9.) $y = \frac{x \sec x}{e^x} \xrightarrow{D}$

$$y' = \frac{e^x \cdot (x \cdot \sec x \tan x + 1 \cdot \sec x) - x \sec x \cdot e^x}{e^{2x}}$$

10.) $y = (\sin x + \cos x)\sec x \xrightarrow{D}$

$$y' = (\sin x + \cos x) \cdot \sec x \tan x + (\cos x - \sin x) \cdot \sec x$$

12.) $y = \frac{\cos x}{1 + \sin x} \xrightarrow{D} y' = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$

13.) $y = \frac{4}{\cos x} + \frac{1}{\tan x} = 4\sec x + \cot x \xrightarrow{D}$

$$y' = 4\sec x \tan x - \csc^2 x$$

15.) $y = (\sec x + \tan x)(\sec x - \tan x) \xrightarrow{D}$

$$y' = (\sec x + \tan x)(\sec x \tan x - \sec^2 x) + (\sec x \tan x + \sec^2 x)(\sec x - \tan x)$$

16.) $y = x^2 \cos x - 2x \sin x - 2\cos x \xrightarrow{D}$

$y' = x^2 \cdot -\sin x + 2x \cos x$
$- (2x \cdot \cos x + 2 \sin x) - 2(-\sin x)$

17.) $f(x) = x^3 \cdot \sin x \cdot \cos x \xrightarrow{D}$

$f'(x) = 3x^2 \cdot \sin x \cdot \cos x + x^3 \cdot \cos x \cdot \cos x$
$+ x^3 \cdot \sin x \cdot -\sin x$

20.) $s = t^2 - \sec t + 5e^t \xrightarrow{D}$

$s' = 2t - \sec t \tan t + 5e^t$

22.) $s = \dfrac{\sin t}{1 - \cos t} \xrightarrow{D}$

$s' = \dfrac{(1-\cos t) \cdot \cos t - \sin t \cdot (\sin t)}{(1-\cos t)^2}$

24.) $r = \theta \sin \theta + \cos \theta \xrightarrow{D}$

$r' = \theta \cdot \cos \theta + (1) \sin x - \sin \theta$

34.) b.) $y = 9 \cos x \longrightarrow$

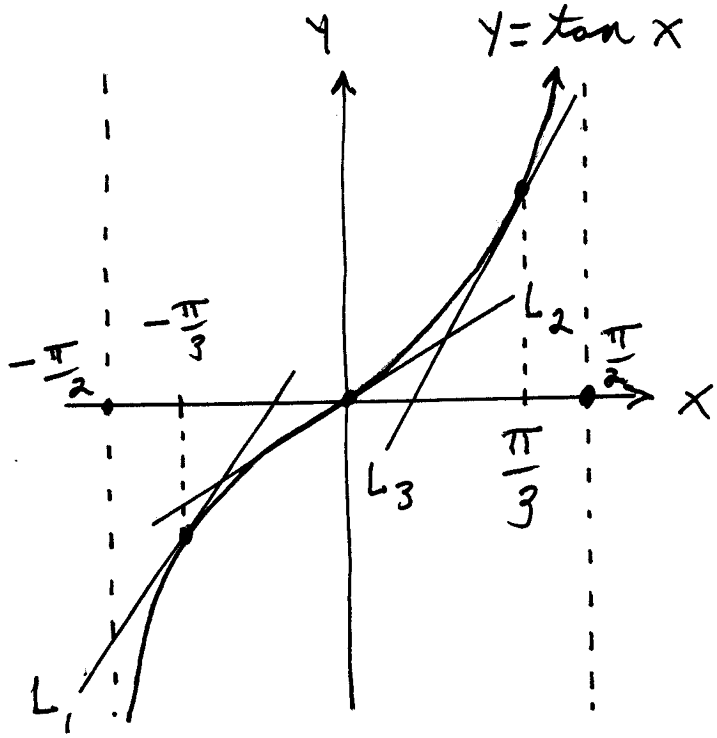
$y' = -9 \sin x \longrightarrow$

$y'' = -9 \cos x \longrightarrow$

$y''' = 9 \sin x \longrightarrow$

$y^{(4)} = 9 \cos x$

36.)



$Y' = \sec^2 x$

a.) $x = -\pi/3 \rightarrow$ slope

$m = Y' = \sec^2\left(-\frac{\pi}{2}\right)$

$= \dfrac{1}{\cos^2(-\pi/3)}$

$= \dfrac{1}{(1/2)^2} = 4$;

$x = -\frac{\pi}{3} \rightarrow Y = \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}/2 \div 1/2 = -\sqrt{3}$;

tangent line is

$L_1: Y - (-\sqrt{3}) = 4(x - -\pi/3) \rightarrow L_1: Y = 4x + \frac{4\pi}{3} - \sqrt{3}$

b.) $x = 0 \rightarrow$ slope $m = Y' = \sec^2(0) = 1$;

$x = 0 \rightarrow Y = \tan 0 = 0$; tangent line is

$L_2: Y - 0 = 1 \cdot (x - 0) \rightarrow L_2: Y = x$

c.) $x = \frac{\pi}{3} \rightarrow$ slope $m = Y' = \sec^2\left(\frac{\pi}{3}\right) = 4$;

$x = \pi/3 \rightarrow Y = \tan(\pi/3) = \sqrt{3}$; tangent line is $L_3: Y - \sqrt{3} = 4(x - \pi/3) \rightarrow$

$L_3: Y = 4x + \sqrt{3} - \frac{4\pi}{3}$

44.) $Y = -x \rightarrow Y' = -1$; $Y = \cot x \rightarrow$

$Y' = -\csc^2 x = \dfrac{-1}{\sin^2 x}$; if $\dfrac{-1}{\sin^2 x} = -1$,

then $\sin^2 x = 1 \rightarrow$ $(0 < x < \pi)$

$\left.\begin{array}{l}\sin x = 1 \\ \text{or } \sin x = -1\end{array}\right\}$ $x = \pi/2$, $y = \cot\frac{\pi}{2} = 0$

46.) $Y = 1 + \sqrt{2}\csc x + \cot x \longrightarrow$

$Y' = -\sqrt{2}\csc x \cot x - \csc^2 x$;

a.) at $(\frac{\pi}{4}, 4)$ slope

$m = Y' = -\sqrt{2}\csc(\frac{\pi}{4})\cot(\frac{\pi}{4}) - \csc^2(\frac{\pi}{4})$

$= -\sqrt{2}\cdot(\sqrt{2})(1) - (\sqrt{2})^2 = -2 - 2 = -4$;

tangent line is $Y - 4 = -4\,(x - \frac{\pi}{4})$.

b.) $Y' = 0 \rightarrow -\sqrt{2}\csc x \cot x - \csc^2 x = 0 \longrightarrow$

$-\csc x \cdot (\sqrt{2}\cot x + \csc x) = 0 \longrightarrow$

$\csc x = \frac{1}{\sin x} = 0$ (impossible) OR

$\sqrt{2}\cot x = -\csc x \rightarrow \sqrt{2}\cdot\frac{\cos x}{\sin x} = \frac{-1}{\sin x} \longrightarrow$

$\sqrt{2}\cos x = -1 \longrightarrow \cos x = -1/\sqrt{2} = -\sqrt{2}/2 \longrightarrow$

$x = \frac{3}{4}\pi \rightarrow Y = 1 + \sqrt{2}\csc(\frac{3}{4}\pi) + \cot(\frac{3}{4}\pi)$

$= 1 + \sqrt{2}\cdot(\sqrt{2}) + (-1) = 2$; tangent line

is $Y = 2$.

47.) $\lim_{x \to 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right) = \sin 0 = 0$

48.) $\lim_{x \to -\frac{\pi}{6}} \sqrt{1 + \cos(\pi \csc x)}$

$= \sqrt{1 + \cos(\pi \cdot \csc(-\frac{\pi}{6}))}$

$= \sqrt{1 + \cos(\pi \cdot (-2))} = \sqrt{1+1} = \sqrt{2}$

52 52.) $\lim\limits_{x \to 0} \sin\left(\dfrac{\pi + \tan x}{\tan x - 2\sec x}\right)$

$= \sin\left(\dfrac{\pi + \tan 0}{\tan 0 - 2\sec 0}\right) = \sin\left(\dfrac{\pi + 0}{0 - 2(1)}\right)$

$= \sin\left(-\dfrac{\pi}{2}\right) = -1$

54 54.) $\lim\limits_{\theta \to 0} \cos\left(\dfrac{\pi\theta}{\sin\theta}\right) = \lim\limits_{\theta \to 0} \cos\left(\pi \cdot \dfrac{1}{\frac{\sin\theta}{\theta}}\right)$

$= \cos(\pi \cdot 1) = -1$

58 58.) $g(x) = \begin{cases} x+b, & \text{if } x < 0 \\ \cos x, & \text{if } x \geq 0 \end{cases}$;

a.) make g continuous at $x = 0$:

i.) $g(0) = \cos 0 = 1$

ii.) $\lim\limits_{x \to 0} g(x)$ must be 1 ! :

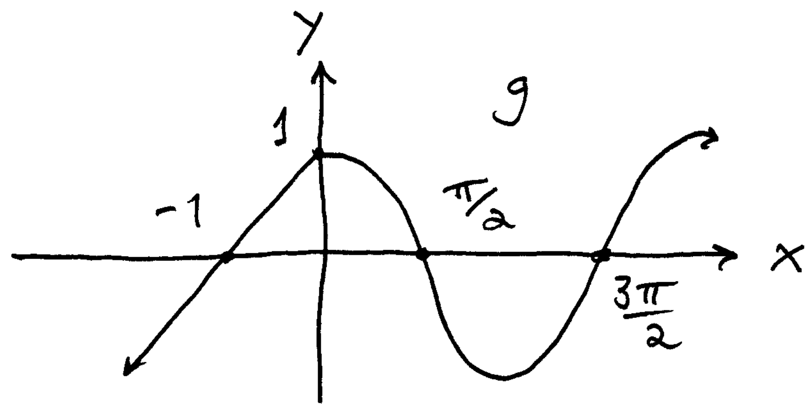
$\lim\limits_{x \to 0^+} g(x) = \lim\limits_{x \to 0^+} \cos x = \cos 0 = 1$;

$\lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^-} (x+b) = 0 + b = b$, so

$\boxed{b = 1}$ and $g(x) = \begin{cases} x+1, & \text{if } x < 0 \\ \cos x, & \text{if } x \geq 0 \end{cases}$;

b.) Is g differentiable at $x = 0$?

$g'(0) = \lim\limits_{h \to 0} \dfrac{g(0+h) - g(0)}{h}$



$$= \lim_{h\to 0} \frac{g(h)-1}{h} \quad ;$$

$$\lim_{h\to 0^+} \frac{g(h)-1}{h} = \lim_{h\to 0^+} \frac{\cos h - 1}{h}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{h\to 0} \frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1}$$

$$= \lim_{h\to 0} \frac{\cos^2 h - 1}{h(\cos h + 1)} = \lim_{h\to 0} \frac{-\sin^2 h}{h(\cos h + 1)}$$

$$= \lim_{h\to 0} -\frac{\sin h}{h} \cdot \frac{\sin h}{\cos h + 1}$$

$$= -(1)\cdot \frac{0}{1+1} = \boxed{0} \quad ;$$

$$\lim_{h\to 0^-} \frac{g(h)-1}{h} = \lim_{h\to 0^-} \frac{(h+\cancel{1})-\cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}} = \lim_{h\to 0} 1 = \boxed{1} \quad ;$$

so $\boxed{g'(0) \text{ DNE}}$!

59.) $\frac{d}{dx}(\cos x) = -\sin x$,

$\frac{d^2}{dx^2}(\cos x) = -\cos x$,

$\rightarrow$ $\frac{d^3}{dx^3}(\cos x) = \sin x$,

$\frac{d^4}{dx^4}(\cos x) = \cos x$; thus

$\frac{d^8}{dx^8}(\cos x) = \cos x$,

$\frac{d^{12}}{dx^{12}}(\cos x) = \cos x$, ...

$\frac{d^{996}}{dx^{996}}(\cos x) = \frac{d^{4\cdot 249}}{dx^{4\cdot 249}}(\cos x) = \cos x$, ...

$\frac{d^{999}}{dx^{999}}(\cos x) = \sin x$

61.) $x = 10\cos t$, $v = x' = -10\sin t$

a.) $x(0) = 10\cos 0 = 10(1) = 10$ cm.

$x\left(\frac{\pi}{3}\right) = 10\cos\frac{\pi}{3} = 10\left(\frac{1}{2}\right) = 5$ cm.

$x\left(\frac{3\pi}{4}\right) = 10\cos\frac{3\pi}{4} = 10\left(-\frac{\sqrt{2}}{2}\right) = -5\sqrt{2}$ cm.

b.) $v(0) = -10\sin 0 = -10(0) = 0$ cm./sec.

$v\left(\frac{\pi}{3}\right) = -10\sin\frac{\pi}{3} = -10\left(\frac{\sqrt{3}}{2}\right) = -5\sqrt{3}$ cm./sec.

$v\left(\frac{3\pi}{4}\right) = -10\sin\frac{3\pi}{4} = -10\cdot\frac{\sqrt{2}}{2} = -5\sqrt{2}$ cm./sec.